

## Cascade fuzzy control of a tubular chemical reactor

Anna Vasičkaninová, Monika Bakošová, Alajos Mészáros

*Slovak University of Technology in Bratislava, Institute of Information Engineering,  
Automation, and Mathematics, Radlinského 9, 81237 Bratislava, Slovak Republic  
[anna.vasickaninova@stuba.sk](mailto:anna.vasickaninova@stuba.sk)*

### Abstract

Cascade control is a multi-loop control structure often used in industrial applications, which offers a possibility for applying advanced controllers. This paper compares cascade control with type-1 fuzzy controllers, type-2 fuzzy controllers, and PID controllers on the case study of a tubular chemical reactor. The primary controllers are type-1 fuzzy PID and PD controllers, type-2 fuzzy PID and PD controllers, or conventional PID controllers. The secondary controllers are type-1 fuzzy P, type-2 fuzzy P or conventional P controllers. Simulation results demonstrate that cascade control with both types of fuzzy controllers can assure better values of followed performance indices and higher energy savings measured by the coolant consumption during control of the tubular chemical reactor.

**Keywords:** cascade control, type-1 fuzzy control, type-2 fuzzy control, PID control, tubular chemical reactor.

### 1. Introduction

Between advanced control strategies, fuzzy logic control is often found in applications where conventional closed loop control does not assure satisfactory results because of non-linearity, asymmetric dynamics, or uncertainties in the controlled processes. Fuzzy logic control is based on the theory of fuzzy sets pioneered by Zadeh (1965). Zadeh (1975) also introduced the concept of the type-2 fuzzy logic. History, application, and possible future of fuzzy control are summarized in Guerra et al. (2015). Mendel (2018) introduced rule-based systems from type-1, interval type-2 and general type-2 fuzzy systems. Mittal et al. (2020) offered overview of past, present, and future trends of type-2 fuzzy logic applications including theoretical and practical implications.

Cascade control is a multiloop control strategy that enables using of advanced controllers. Meng and Hou (2011) designed cascade control with main fuzzy PID controller and auxiliary PID controller for hydro-viscous drive speed regulating start. Kumbasar and Hagraš (2013) proposed a cascade control architecture, which includes the inner and outer control loops for the path tracking control of mobile robots in presence of uncertainty. Garcia et al. (2007) designed fuzzy logic controller with intermediate variable as an alternative for cascade control with fuzzy controllers and compared both strategies. Xie and Liu (2017) formed fuzzy cascade control based on known control history for superheated temperature.

Despite intensive research and promising applications in various fields, there is a lack of studies devoted to implementation of type-1 fuzzy logic controllers (T1FLCs) and type-2 fuzzy logic controllers (T2FLCs) to tubular chemical reactors and T2FLCs in cascade control. The main goal of this paper is to show that cascade control (CC) with T2FLCs can guarantee energy savings and better performance compared to CC with conventional

PID controllers and is alternative to CC with T1FLCs when controlling systems with uncertainties, asymmetric dynamics or nonlinear systems, as tubular chemical reactors.

## 2. Cascade fuzzy control

### 2.1. Cascade control

Cascade control (CC) (Figure 1) is a multi-loop control structure used in process industry to improve control under immeasurable disturbances (Bequette, 2003). In Figure 1,  $C_1$  is the primary (main) controller,  $C_2$  is the secondary (auxiliary) controller,  $P_1$  is the primary controlled system, and  $P_2$  is the secondary controlled system. Signals  $r_1$  and  $r_2$  represent reference values,  $y_1$  and  $y_2$  are controlled outputs,  $e_1$  and  $e_2$  are errors,  $u_2$  is the manipulated variable that results from the control input calculated by  $C_2$  influenced by the disturbance  $d_2$ . The disturbance  $d_1$  influences the primary controlled output  $y_1$ . Both controllers in the CC can be fuzzy controllers.

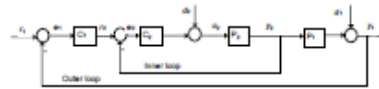


Figure 1: Scheme of a cascade control system

### 2.2. Type-1 fuzzy control and interval type-2 fuzzy control

The structure of T1FLC is represented in Figure 2. The crisp inputs to the dynamic controller can be errors, derivatives of errors, integrals of errors or previous values of measurements backward in time. Fuzzifier converts input data to degrees of membership by a lookup in one or several membership functions. Rule base includes various empirical rules. Defuzzifier converts the resulting fuzzy set to numbers that enter the process as control inputs.

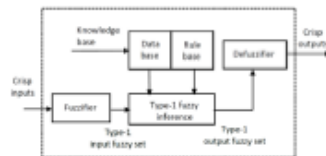


Figure 2: Type-1 fuzzy controller

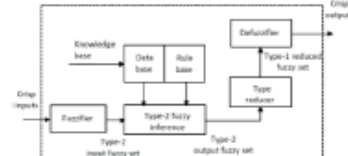


Figure 3: Interval type-2 fuzzy controller

Figure 3 represents structure of T2FLC. The rule base for T2FLC remains the same as for T1FLC, but its membership functions are type-2 interval fuzzy sets, and a reducer must be used prior defuzzification (Kumbasar, 2014). The advantage of using type-2 fuzzy logic (FL) compared to type-1 FL is that type-2 FL can handle uncertainty in control, which may be due to noise, dynamic changes in the environment, or imprecision in the models (Mittal et al., 2020).

## 3. Case study

The case study from chemical engineering domain is devoted to a tubular chemical reactor (TCR) with exothermic consecutive reactions  $A \xrightarrow{k_1} B \xrightarrow{k_2} C$  in the liquid phase and with the co-current cooling (Dostal et al., 2015). Vasičkaninová et al. (2019) did steady-state analysis and step-response based identification of the TCR and based on presented results, TCR is the nonlinear system with asymmetric dynamics and can be treated as a system with uncertainty. As B is the main product and C is the side product, it is necessary to

keep the concentration  $c_B$  at the reference value. In CC of TCR, the concentration  $c_B$  is the primary controlled output and the temperature of reaction mixture  $T$  is the secondary controlled output. The manipulated variable is the flow rate of coolant  $q$ .

3.1. Cascade control of the tubular chemical reactor using conventional PID controllers  
The transfer function of the PID controller has the form (Mikleš and Fikar, 2007)

$$C = k_p \left( 1 + \frac{1}{t_i s} + t_d s \right) \tag{1}$$

where  $k_p$  is the proportional gain,  $t_i$  is the integral time,  $t_d$  is the derivative time. The secondary controller was tuned experimentally as a P controller. The primary PID controller was tuned using the Rivera-Morari method (PID-RM) and the primary PI controller was tuned using the Cohen-Coon method (PI-CC) (Bequette, 2003). Two primary controllers assuring best simulation results were chosen from several designed controllers and no fine-tuning was done. Table 1 presents the controller parameters.

Table 1: PID controller parameters

Controller parameters	Primary controllers		Secondary controller
	PID-RM	PI-CC	P
$k_p$	1.95	2.18	-0.4
$t_i$	14.30	3.86	
$t_d$	0.67		

3.2. Cascade control of the tubular chemical reactor using fuzzy controllers

3.2.1. Secondary type-1 fuzzy P controller and secondary type-2 fuzzy P controller

Both, the secondary type-1 fuzzy P controller (P-T1FLC) and the secondary type-2 fuzzy P controller (P-T2FLC) were designed as Sugeno-type fuzzy inference systems (FISs), each with 2 rules

$$\text{If } e \text{ is } A_i \text{ Then } f_i = p_i e + q_i \tag{2}$$

where  $e$  is the error,  $p_i, q_i$  are the consequent parameters presented in Table 2 together with the antecedent parameters  $A_i$  and the parameters of the symmetric Gaussian membership function  $\sigma_i, c_i$  (Zhao and Bose, 2002) used for fuzzification of inputs.

Table 2: Parameters of symmetric Gaussian functions, antecedent and consequent parameters

Rule	$\sigma_i$	$c_i$	$A_i$	$p_i$	$q_i$
1	5.93	-14.79	$A_1$	-0.029	0.23
2	5.93	-0.81	$A_2$	-0.031	0.24

3.2.2. Primary type-1 fuzzy PD controller and primary type-2 fuzzy PD controller

Both, the primary type-1 fuzzy PD controller (PD-T1FLC) and the primary type-2 fuzzy PD controller (PD-T2FLC) were designed as the Sugeno-type FISs with 6 rules:

$$\text{If } e \text{ is } A_i \text{ and } \frac{de}{dt} \text{ is } B_i \text{ Then } f_i = p_i e + q_i \frac{de}{dt} + r_i \tag{3}$$

where  $e$  is the error,  $de/dt$  is the derivative of error,  $A_i, B_i$  are the antecedent parameters and  $p_i, q_i, r_i$  are the consequent parameters, which are presented in Table 3.

Sugeno-type FISs were generated using the subtractive clustering method. Triangular membership functions (Zhao and Bose, 2002) were used for fuzzification of inputs and Table 4 presents parameters of used triangular membership functions.

Table 3: Antecedent parameters and consequent parameters

Rule	$A_i$	$B_i$	$p_i$	$q_i$	$r_i$
1	$A_1$	$B_1$	121.29	-16.33	316.04
2	$A_1$	$B_2$	-22.92	-68.84	329.37
3	$A_2$	$B_1$	87.13	-192.30	201.02
4	$A_2$	$B_2$	30.52	128.56	370.74
5	$A_3$	$B_1$	103.65	-28.07	142.95
6	$A_1$	$B_3$	84.70	-4.02	178.70

Table 4: Parameters of triangular membership functions

$e$			$de/dt$		
$a_i$	$b_i$	$c_i$	$a_i$	$b_i$	$c_i$
-0.96	-0.05	0.86	-0.59	-0.28	0.006
-0.01	0.86	1.78	-0.28	0.02	0.32
0.87	1.78	2.70			

### 3.2.3. Primary type-1 fuzzy PID controller and primary type-2 fuzzy PID controller

Both, the primary type-1 fuzzy PID controller (PID-T1FLC) and the primary type-2 fuzzy PID controller (PID-T2FLC) were designed as the Sugeno-type FISs with 8 rules:

$$\text{If } e \text{ is } A_i \text{ and } \frac{de}{dt} \text{ is } B_i \text{ and } \int e dt \text{ is } C_i \text{ Then } f_i = p_i e + q_i \frac{de}{dt} + r_i \int e dt + s_i \quad (4)$$

where  $e$  is the error,  $de/dt$  is the derivative of error,  $\int e dt$  is the integral of error,  $p_i$ ,  $q_i$ ,  $r_i$ ,  $s_i$  are the consequent parameters. Table 5 presents the antecedent and consequent parameters. Table 6 shows the parameters of the symmetric Gaussian membership functions (Zhao and Bose, 2002) used for the fuzzification of inputs.

Table 5: Antecedent and consequent parameters

Rule	$A_i$	$B_i$	$C_i$	$p_i$	$q_i$	$r_i$	$s_i$
1	$A_1$	$B_1$	$C_1$	52.44	-0.41	132.53	45.48
2	$A_1$	$B_1$	$C_2$	25.95	11.54	31.96	52.56
3	$A_1$	$B_2$	$C_1$	27.66	-0.40	0.18	330.32
4	$A_1$	$B_2$	$C_2$	-2.44	0.07	0.15	330.53
5	$A_2$	$B_1$	$C_1$	129.66	91.08	21.09	111.06
6	$A_2$	$B_1$	$C_2$	141.61	-18.23	6.13	66.79
7	$A_2$	$B_2$	$C_1$	144.14	-2.60	11.26	85.82
8	$A_2$	$B_2$	$C_2$	13.47	-2.26	50.51	16.70

Table 6: Parameters of symmetric Gaussian membership functions

$e$		$de/dt$		$\int e dt$	
$\sigma_i$	$c_i$	$\sigma_i$	$c_i$	$\sigma_i$	$c_i$
0.82	-0.04	0.06	-0.26	3.12	0.17
0.59	1.83	0.05	0.07	3.07	7.29

## 4. Simulation results

The MATLAB/Simulink R2021b programming environment was exploited for simulations using CPU i7-11700 2.50 GHz, 32 GB RAM. The simulation results for six scenarios in reference tracking and disturbance rejection are presented in Figures 4 and 5.

The primary reference value was the desired value of the product concentration  $c_B = 2.15 \text{ kmol m}^{-3}$ . The disturbances were represented by increasing the flow rate of the reaction mixture from  $0.2$  to  $0.36 \text{ m}^3\text{s}^{-1}$  at time  $100 \text{ s}$  and then by decreasing to  $0.1 \text{ m}^3\text{s}^{-1}$  at time  $200 \text{ s}$ . The results were compared numerically assessing the total consumption of cooling water  $V$  during control, the integral performance index IAE (integrated absolute error), and ISE (integrated squared error) defined e. g. in Mikleš and Fikar (2007). Table 7 summarizes these numerical results.

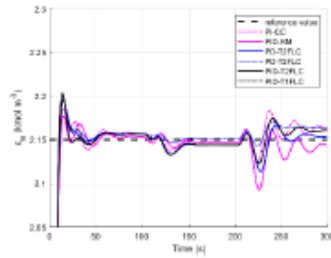


Figure 4: Control responses of the product concentration  $c_B$  in six scenarios

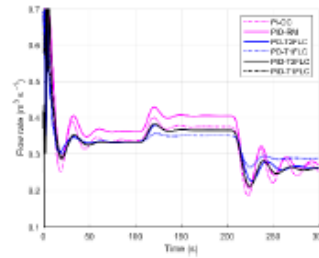


Figure 5: Trajectories of the manipulated variable in six scenarios

Table 7: Values of  $V$ , IAE, and ISE

Scenario	Primary controller	Secondary controller	$V$ ( $\text{m}^3$ )	IAE ( $\text{kmol m}^{-3} \text{ s}$ )	ISE ( $\text{kmol}^2 \text{ m}^{-6} \text{ s}$ )
1	PI-CC	P	100.4396	9.9652	8.4881
2	PID-RM	P	108.3361	9.7175	8.5058
3	PD-T1FLC	P-T1FLC	99.7704	9.5105	8.4505
4	PD-T2FLC	P-T2FLC	99.2400	9.6808	8.4572
5	PID-T1FLC	P-T1FLC	98.5960	9.9505	8.4576
6	PID-T2FLC	P-T2FLC	98.5958	9.7361	8.3604

CC with PID-T2FLC and P-T2FLC guaranteed the lowest coolant consumption and the coolant consumption using CC with PID-T1FLC and P-T1FLC was almost the same. The coolant consumption increased by 0.65 % if CC with PD-T2FLC and PD-T2FLC was used and by 1.19 % for CC with PD-T1FLC and P-T1FLC. CC with conventional PID-RM and P assured the highest coolant consumption. CC with PD-T1FLC and P-T1FLC reached the lowest value of IAE and the second best according to IAE was CC with PD-T2FLC and P-T2FLC. The worst cascade control according to IAE achieved conventional CC with PI-CC and P controllers. The IAE increased in this CC by 4.78 %. The best value of ISE assured CC with PID-T2FLC and P-T2FLC. The second best was the CC with PD-T1FLC and P-T1FLC with the ISE value greater by 1.08%. CC with PID-RM and P controller was the worst with the ISE value higher by 1.74% compared to the best CC with PID-T2FLC and P-T2FLC. Comparing the coolant consumption, the ISE and IAE values, the CC with the primary PID-T2FLC and the secondary P-T2FLC was the best CC scenario.

**5. Conclusions**

CC with conventional controllers, type-1 fuzzy controllers, and type-2 fuzzy controllers was studied on TCR. CC with the primary PID-T2FLC and the secondary P-T2FLC



assured the most efficient operation of TCR. This scenario assured the lowest coolant consumption and the lowest value of the ISE performance index. According to the IAE performance index, CC with the primary PD-T1FLC and the secondary P-T1FLC was the best. The second best was CC with the primary PD-T2FLC and the secondary P-T2FLC. Based on the comparison of all results, it can be stated that both types of FLCs can be used successfully in cascade control for reaching the goals of control. Application of more complicated fuzzy type-2 controllers helped to improve the energetic efficiency of the studied TCR measured by coolant consumption. Further intensive research in this field will continue in the future.

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