Set-Membership Estimation using Ellipsoidal Ensembles

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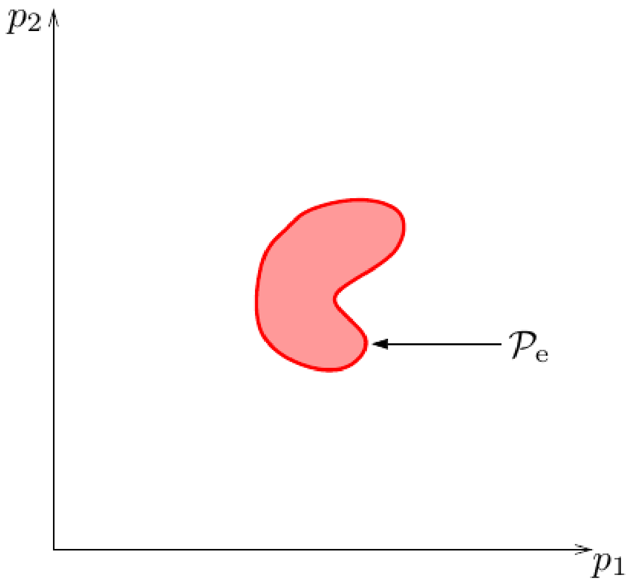
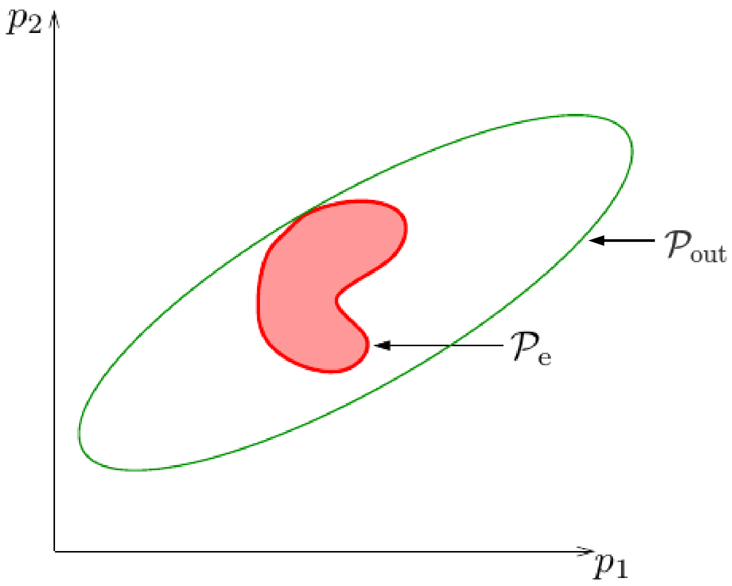
Motivation

- Set-membership estimation is an alternative to statistical estimation
- It drops the need of knowing statistical distributions but requires set-based tools



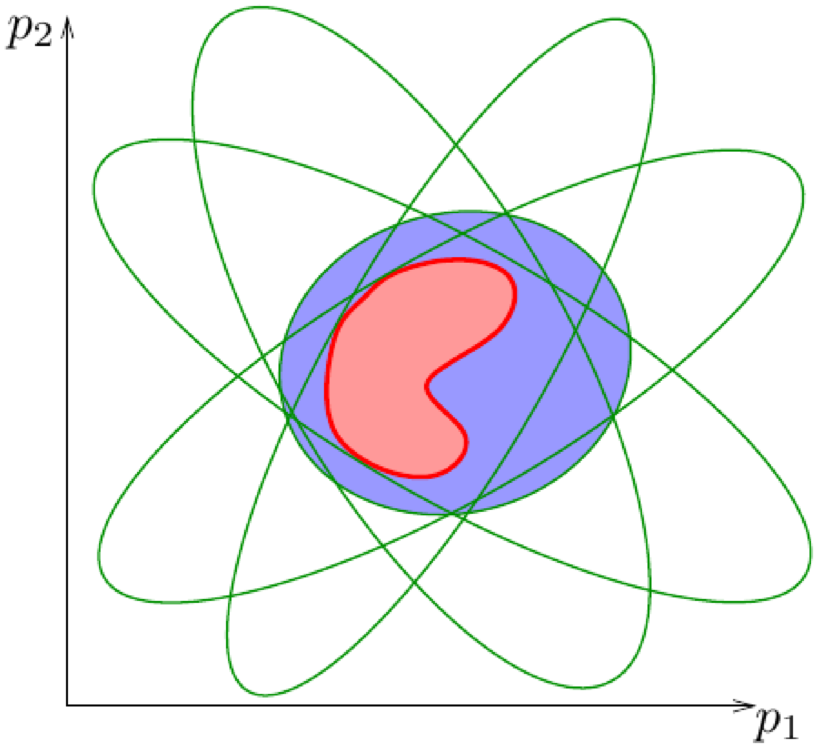
$$\mathcal{P}_\mathrm{e} := \{\boldsymbol{p} \in \mathcal{P}_0 \subset \mathbb{R}^n | \boldsymbol{h}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{p}) \leq 0\}$$



$$\mathcal{P}_\mathrm{e} \subseteq \mathcal{P}_\mathrm{out}$$

This talk

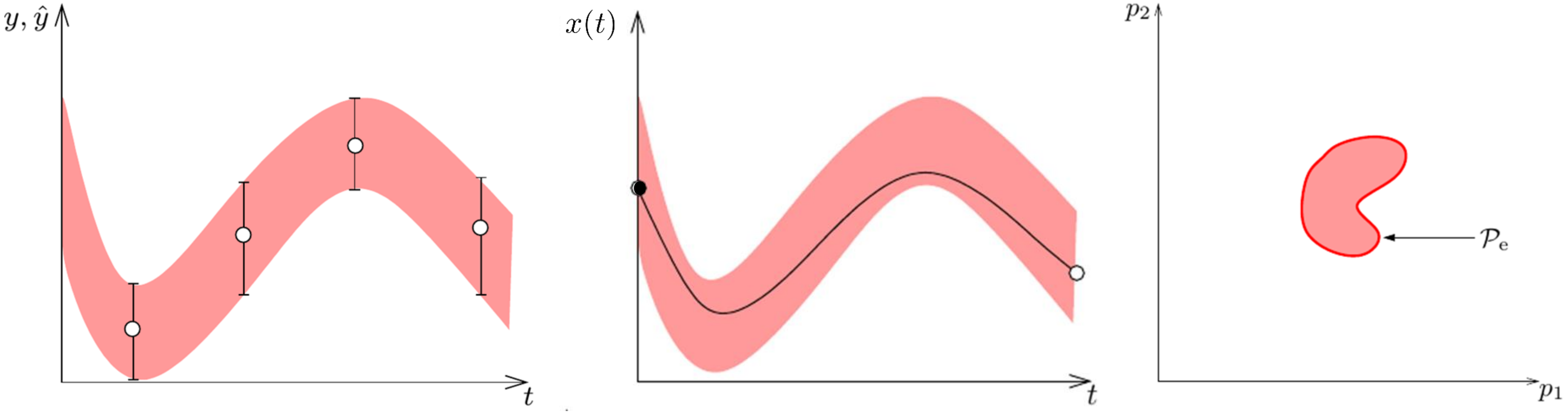
- A novel parameter/state set-membership estimation algorithm for nonlinear systems based on ellipsoidal ensembles that enables online applications



Set-membership estimation (SME)

dynamic system: $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{p}),\ \boldsymbol{x}(0) = \boldsymbol{x}_0(\boldsymbol{p})$ → Set propagation

output function: $\hat{\boldsymbol{y}}(t_i) = \boldsymbol{g}(\boldsymbol{x}(t_i), \boldsymbol{p})$ → Set interesection



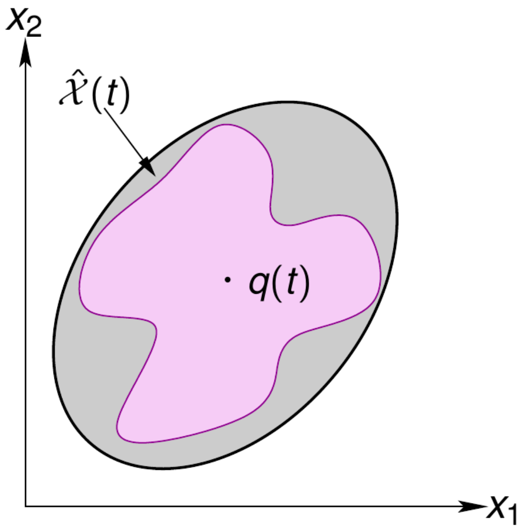
$$\mathcal{P}_e := \{\boldsymbol{p} \in \mathcal{P}_0 | \boldsymbol{h}(\dot{\boldsymbol{x}}, \boldsymbol{x}, \boldsymbol{p}) \leq 0\}$$

Reachable set of a dynamic system

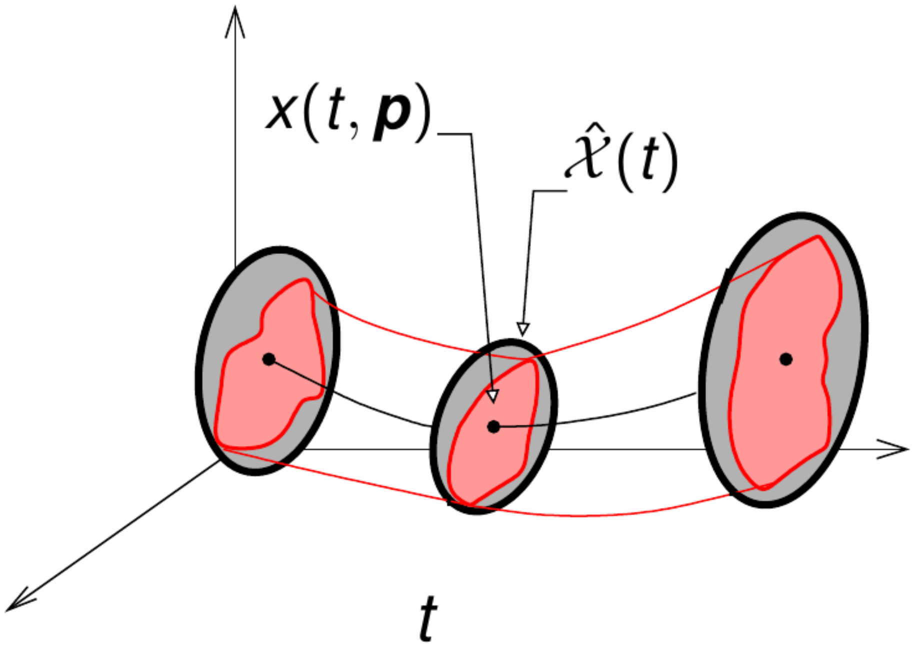
- Given a parameter set $\mathcal{P}$ and a set of parametric ODEs, identify the set $\mathcal{X}(t)$

$$\hat{\mathcal{X}}(t) \subset \mathcal{X}(t) \ni \boldsymbol{x}(t, \boldsymbol{p}) \quad \forall \boldsymbol{p} \in \mathcal{P}, \ \forall t \geq 0$$

- Idea: Approximate the reachable set as an ellipsoidal tube



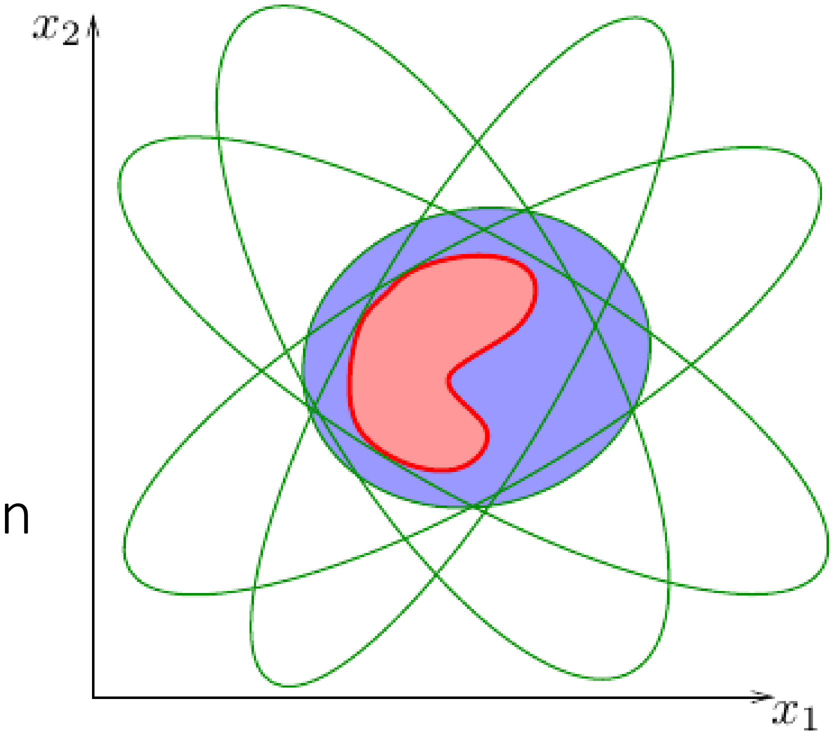
$$\hat{\mathcal{X}}(Q(t), q(t)) := \left\{ Q(t)^{\frac{1}{2}} v + q(t) \mid v^T v \leq 1 \right\}$$



Kurzhanski and Varaiya, Dynamics of Continuous, Discrete & Impulsive System, 2002
Houska, et al. Journal of Process Control 2012.

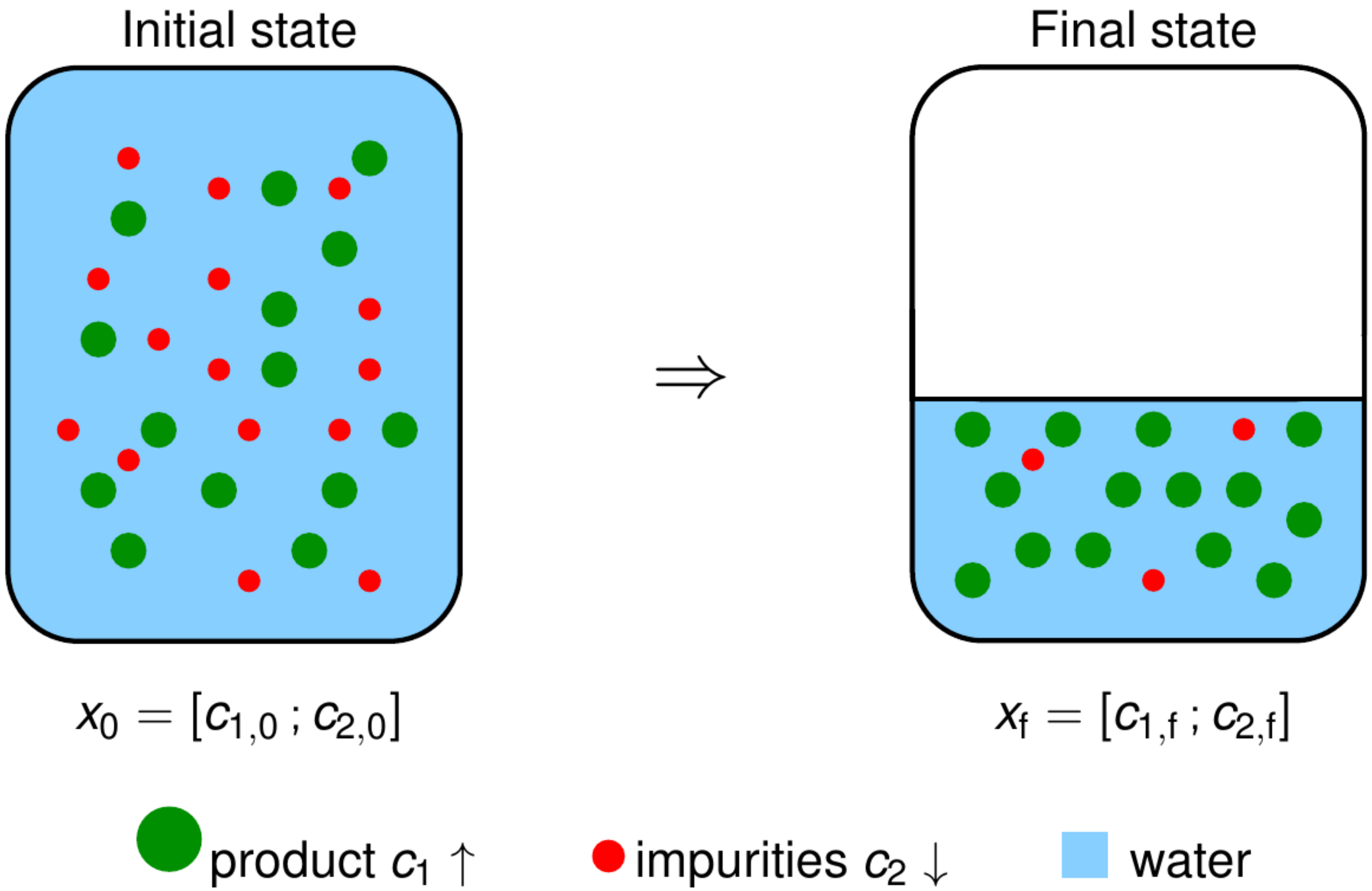
SME with ellipsoidal ensembles

- Idea: Each uncertain measurement point can be projected onto the state space and over-approximated as an ellipsoid
  - Usual tricks can be used: dummy states/parameters, degenerate ellipsoids, etc.
- Idea #2: Ellipsoidal ensemble can be propagated and contracted towards the intersection all other ellipsoids
  - This can be formulated as a set propagation much like the reachable set over-approximation
  - The estimation can be formulated as a dynamic optimization problem -> NLP

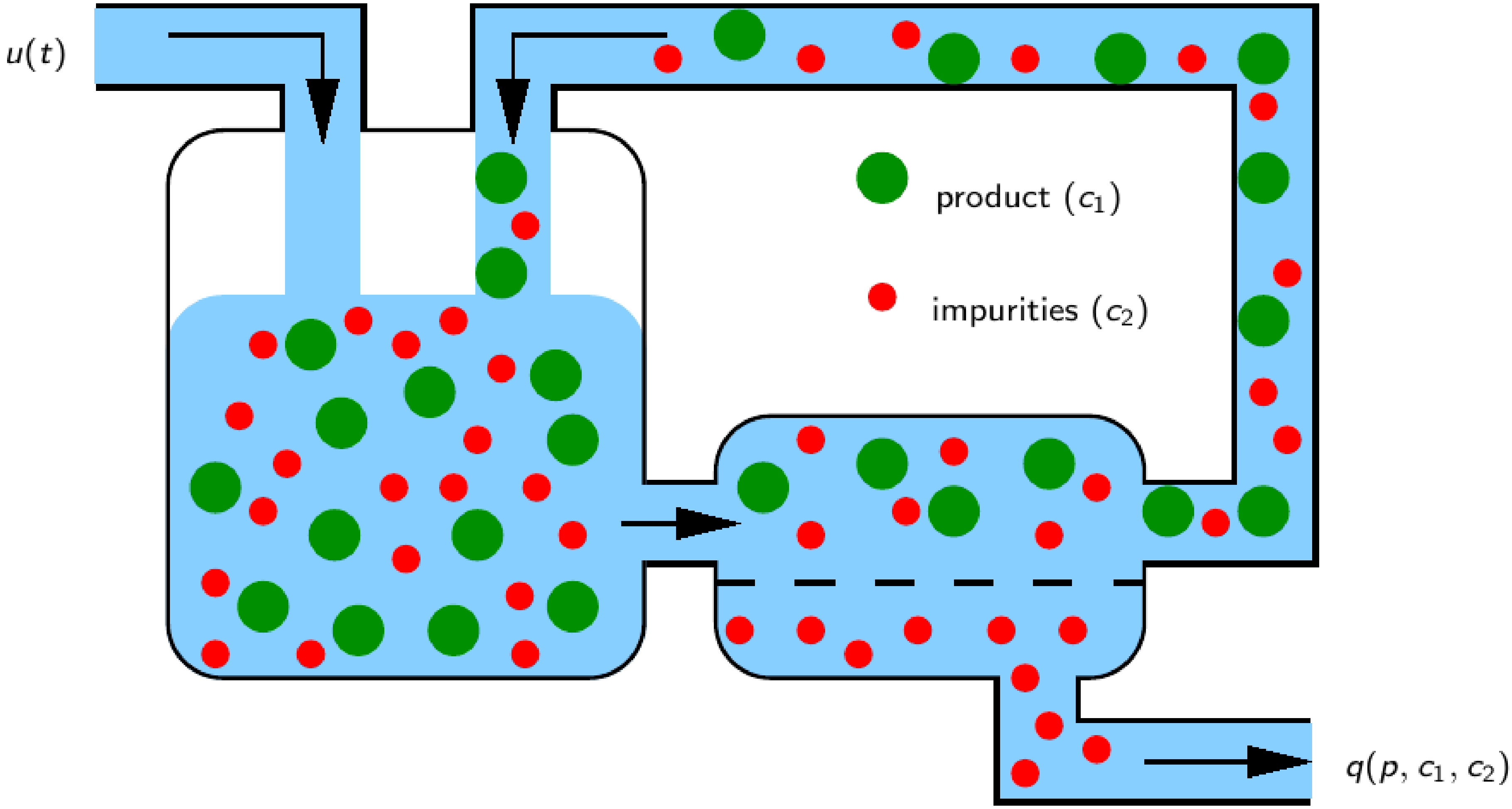


Feng et al., Automatica, 2020

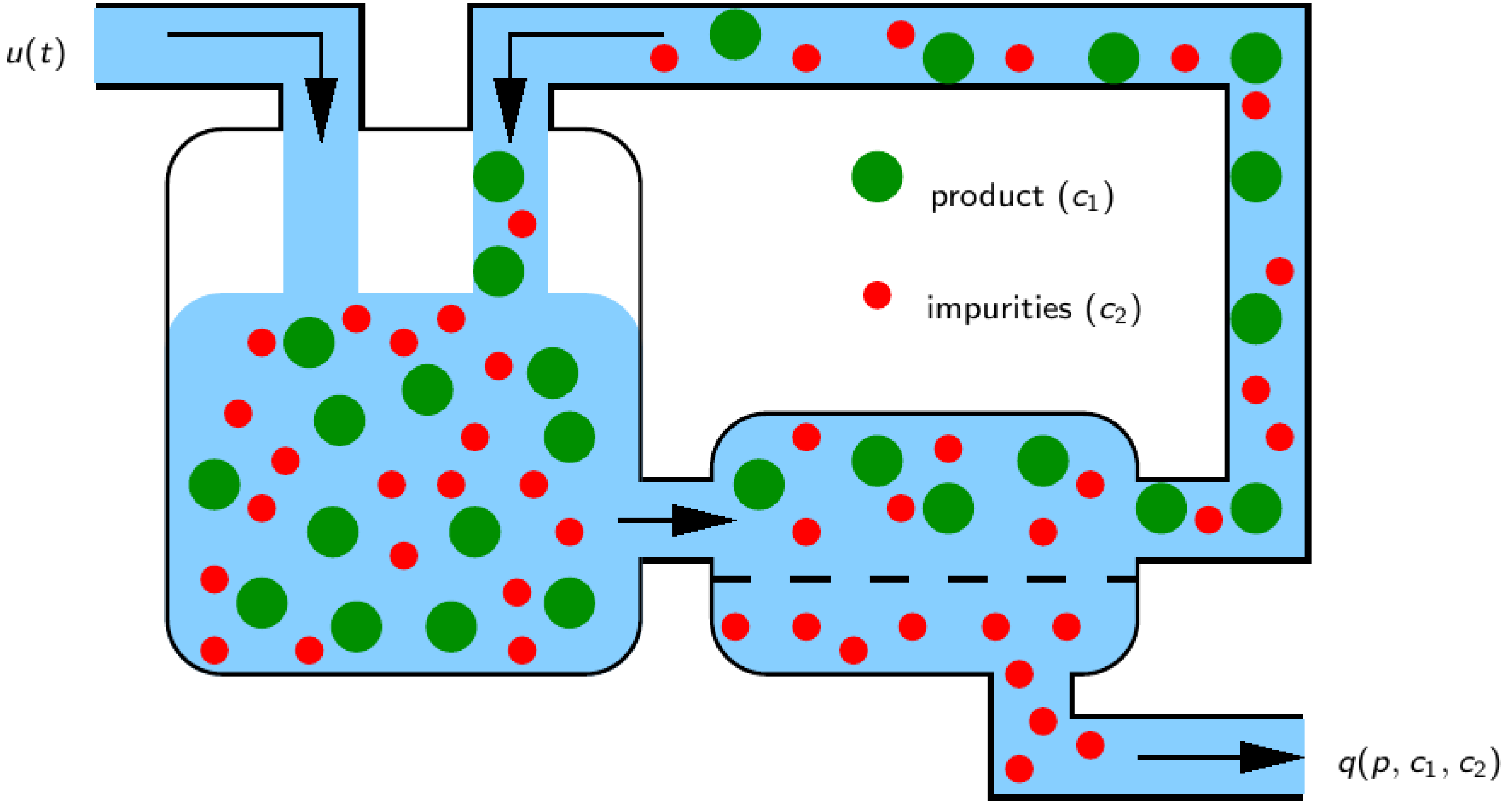
Membrane separation – Goal

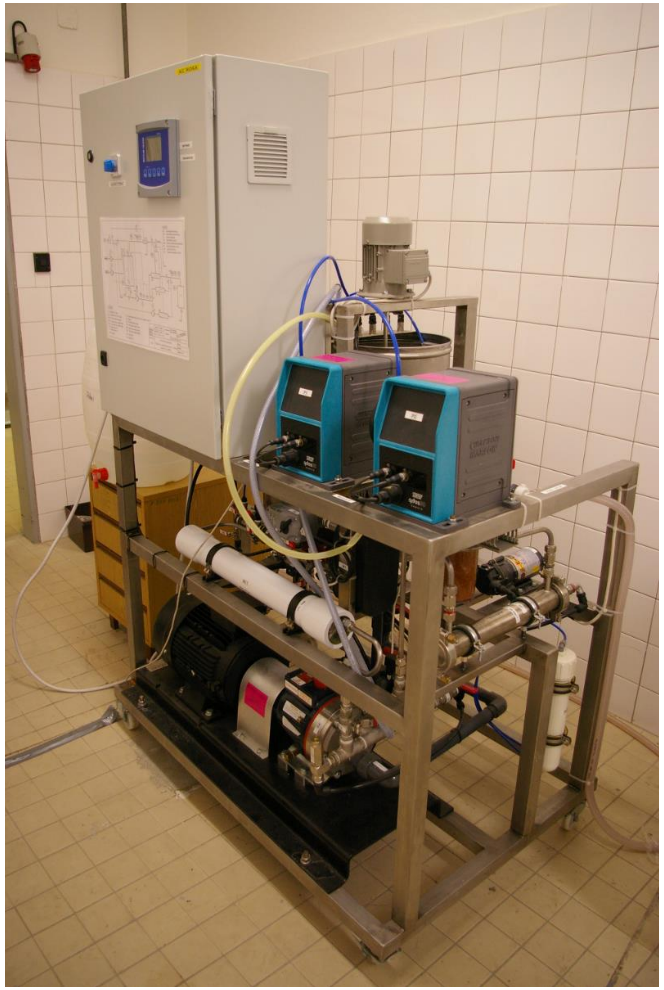


Membrane separation – Process

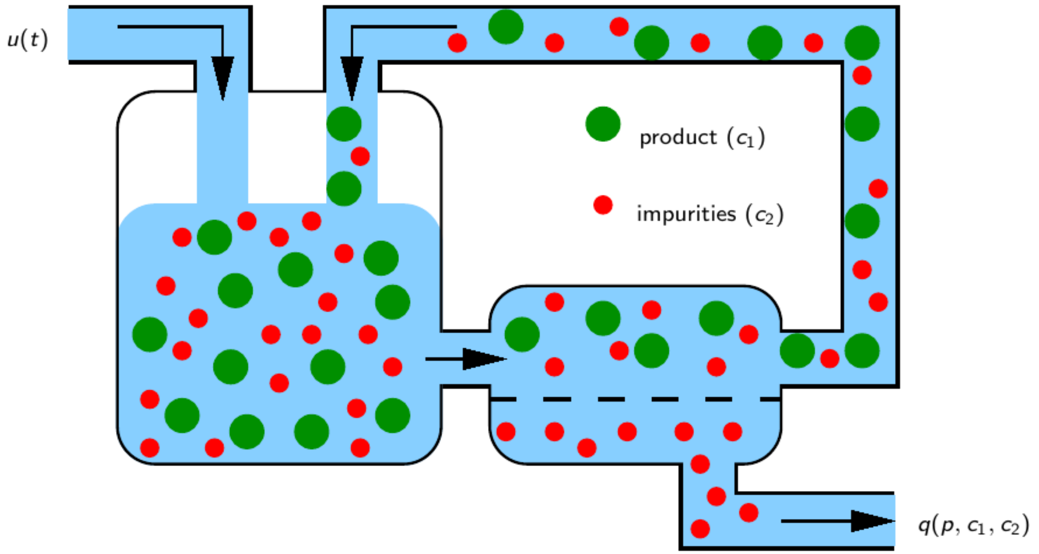


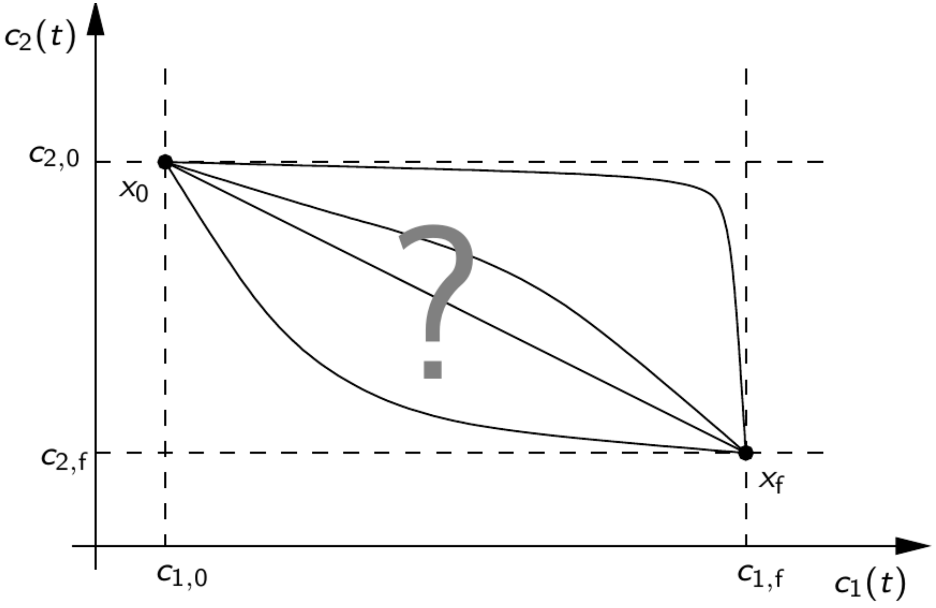
Membrane separation – Plant





Membrane separation – Performance

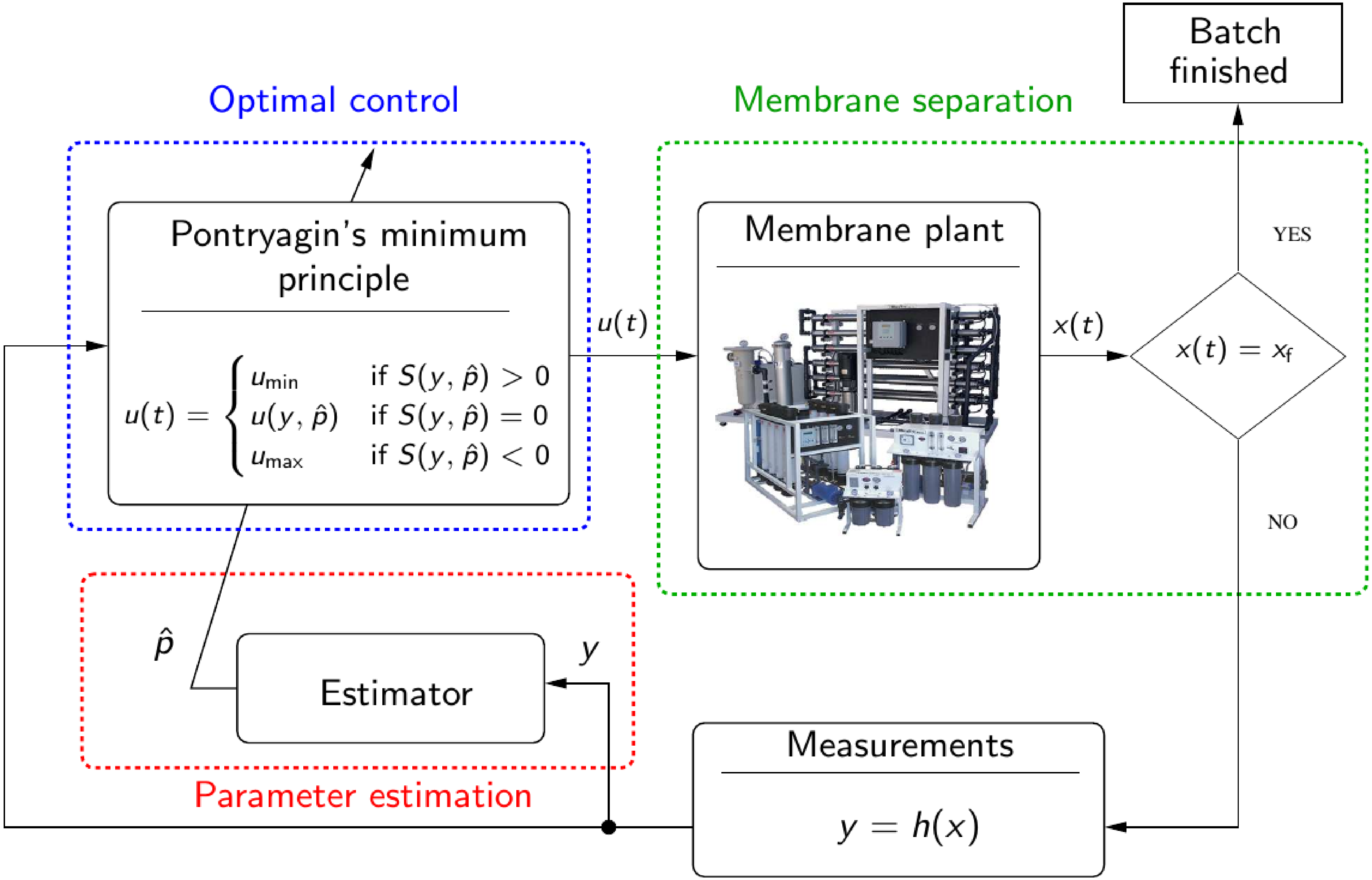




$$u(t) = \begin{cases} u_{\min} & \text{if } S(\boldsymbol{y}, \hat{\boldsymbol{p}}) > 0 \\ u(\boldsymbol{y}, \hat{\boldsymbol{p}}) & \text{if } S(\boldsymbol{y}, \hat{\boldsymbol{p}}) = 0 \\ u_{\max} & \text{if } S(\boldsymbol{y}, \hat{\boldsymbol{p}}) < 0 \end{cases}$$

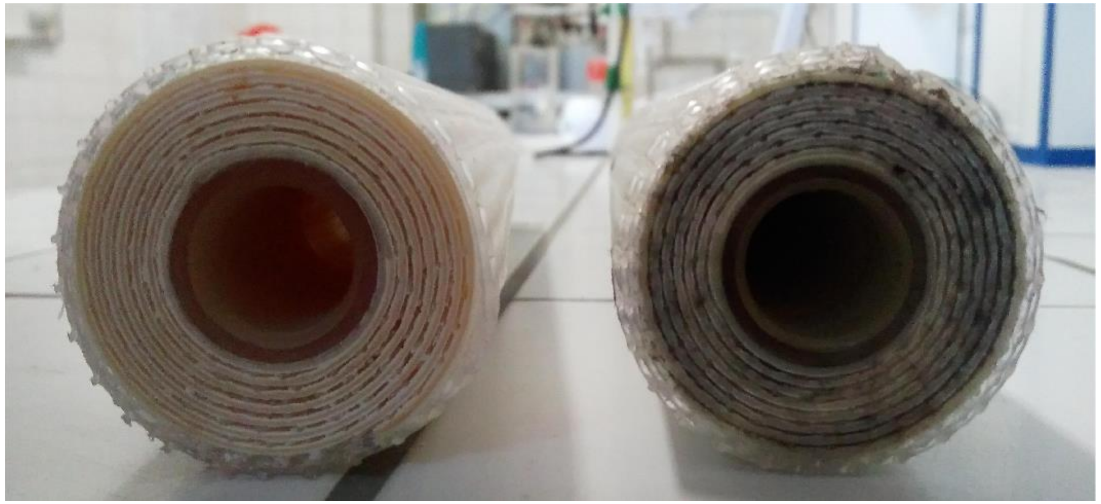
$$x_0 \to x_f \quad \Leftrightarrow \quad \begin{cases} \text{product } c_1 \uparrow \\ \text{impurities } c_2 \downarrow \end{cases}$$

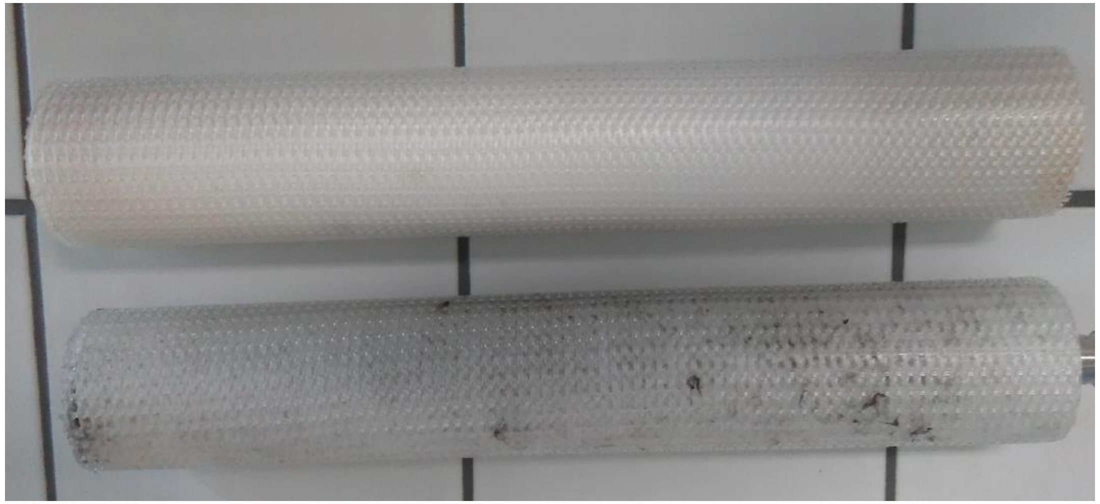
Paulen et al., Journal of Process Control, 2015



Optimal control
Membrane separation
Batch finished
Pontryagin's minimum principle
$u(t) = \begin{cases} u_{min} & \text{if } S(y, \hat{p}) > 0 \\ u(y, \hat{p}) & \text{if } S(y, \hat{p}) = 0 \\ u_{max} & \text{if } S(y, \hat{p}) < 0 \end{cases}$
$u(t)$
Membrane plant
$x(t)$
$x(t) = x_f$
YES
NO
$\hat{p}$
Estimator
y
Measurements
$y = h(x)$
Parameter estimation

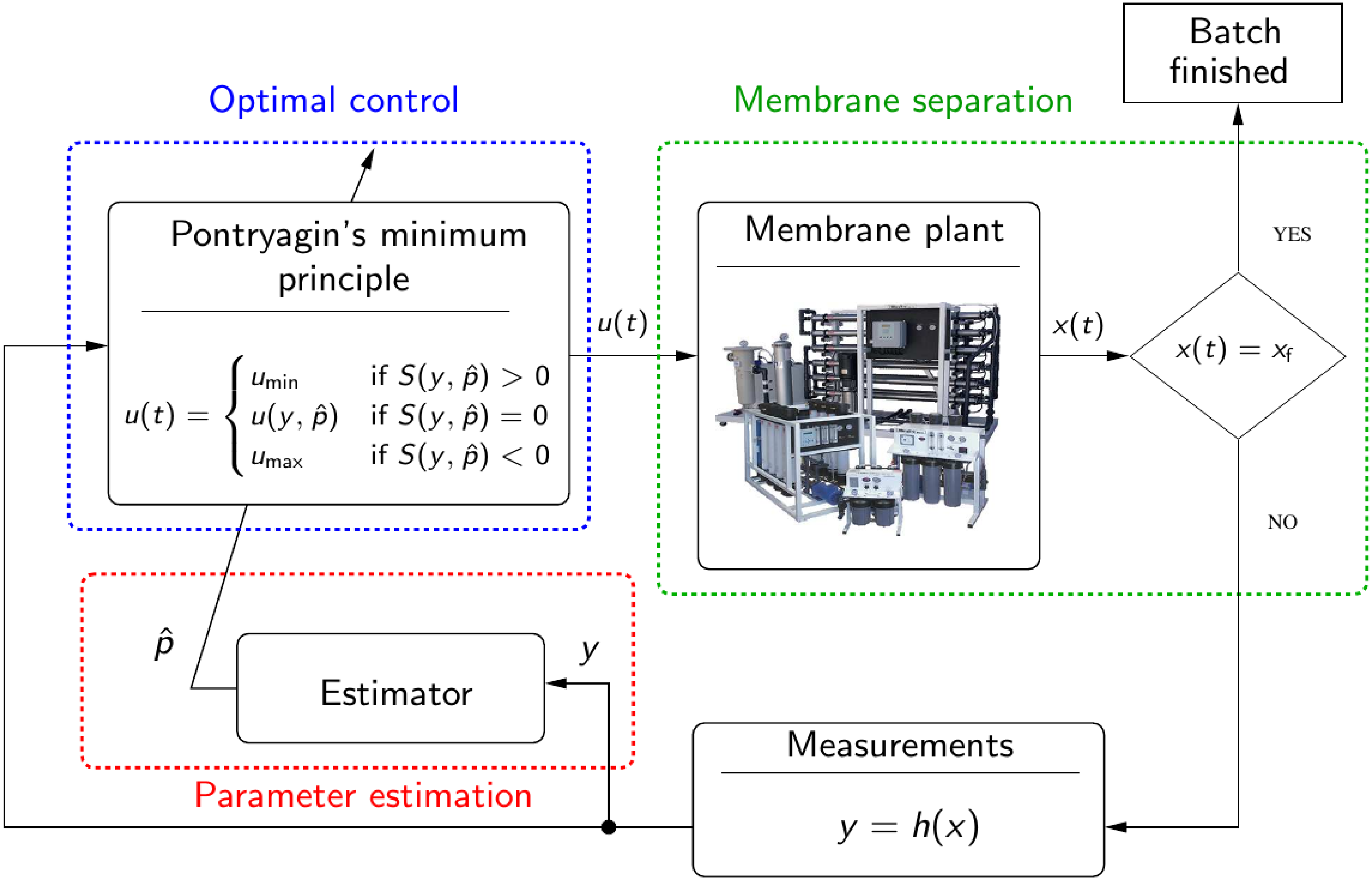
Membrane separation – Fouling





- Gradual fouling of the membrane changes its permeation properties and causes batch-to-batch variations
  - Batch optimality can be restored if a reliable online parameters estimate is in place

Jelemenský et al., Processes, 2016; Paulen and Fikar, Computers & Chemical Engineering, 2019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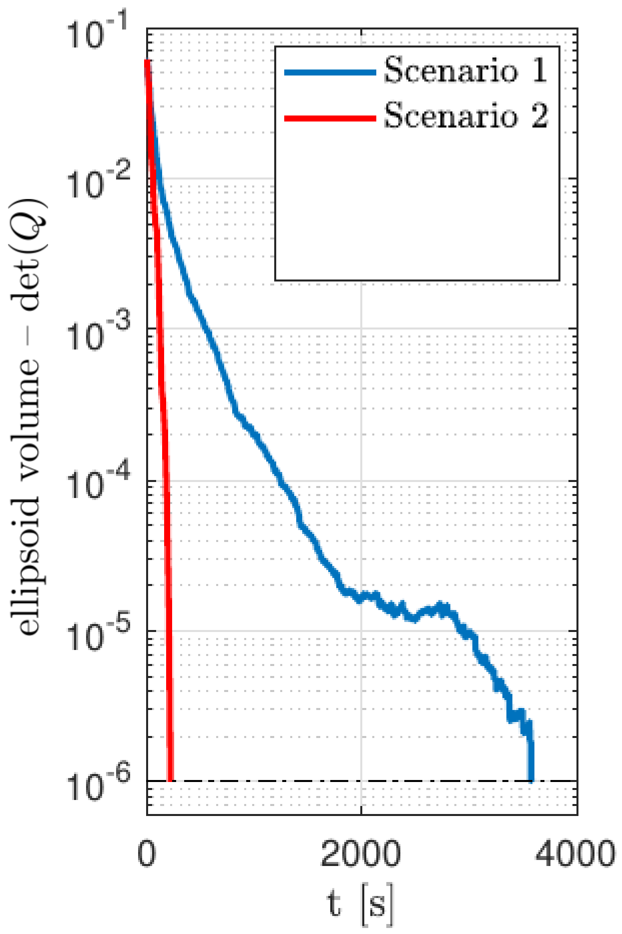
$u(t)$
Membrane plant
$x(t)$
$x(t) = x_f$
YES
NO
$\hat{p}$
Estimator
y
Parameter estimation
Measurements
$y = h(x)$

Performance. How many tubes to use?

- Trade-off between accuracy and complexity

$$\uparrow N \rightarrow \downarrow \det(Q) \text{ vs. } \uparrow \text{CPU time}$$

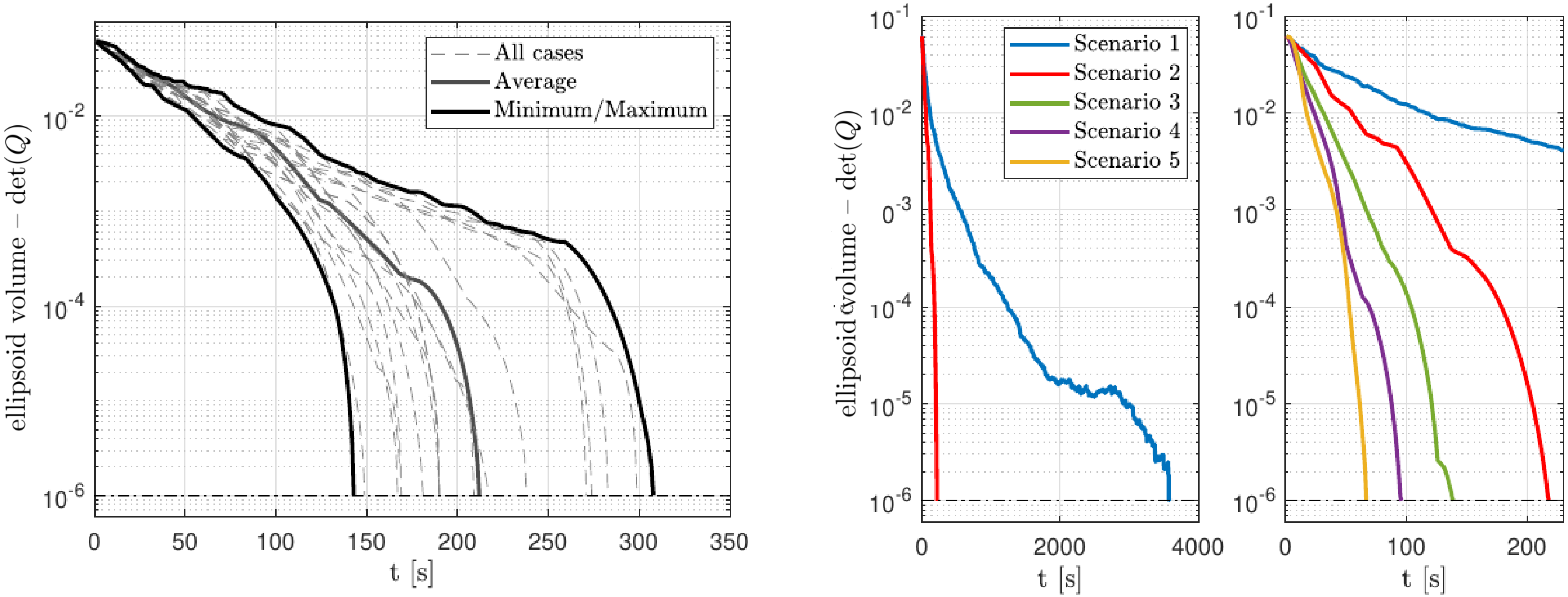
	N	Tube selection	CPU [s]
Scenario 1	2	✗	0.2



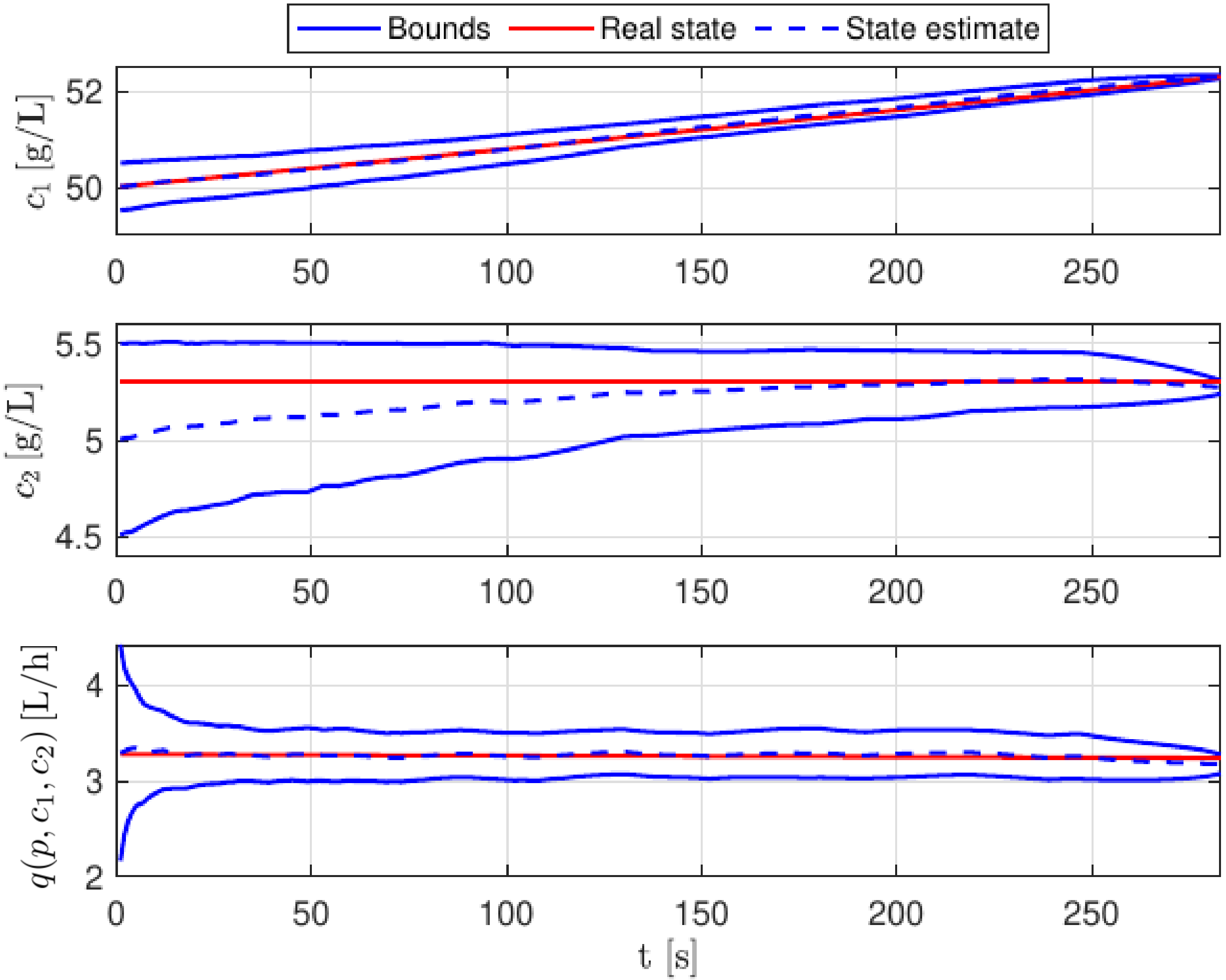
*CasADi, Matlab, standard PC, no parallelization/code optimization

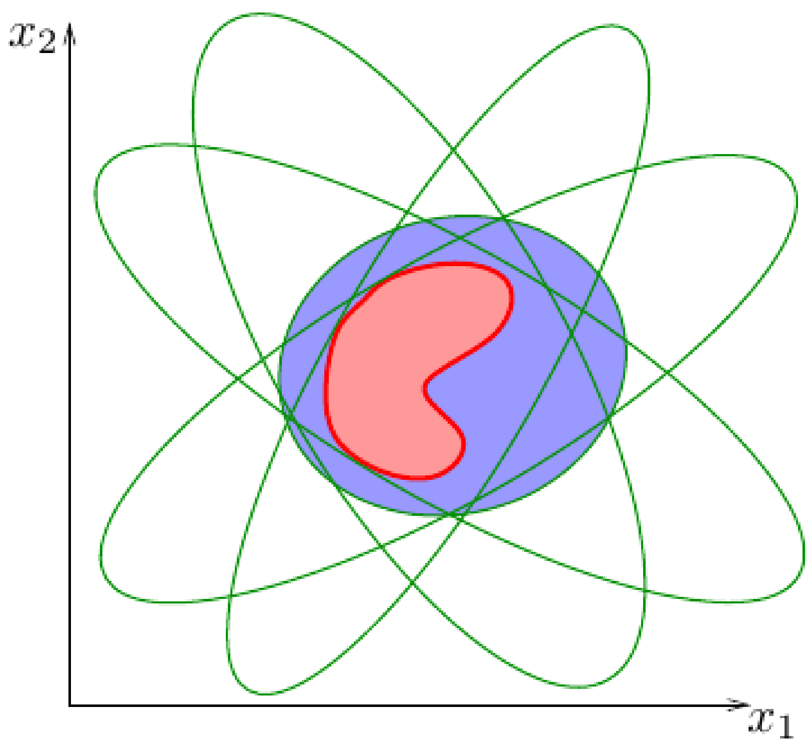
Estimation performance – Scenario 2

- Consistent estimation performance



Estimation performance – Scenario 2





Conclusions

- Novel approach for set-membership state/parameter estimation for nonlinear dynamic systems
  - online applicable, real-time feasible for moderate-scale systems
  - compatible with moving-horizon principle
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