SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

FACULTY OF CHEMICAL AND FOOD TECHNOLOGY

Reg. No.: FCHPT-19990-50920

MPC-based Reference Governors: Theory and Applications

DISSERTATION THESIS

2017

Ing. Martin Klaučo

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

FACULTY OF CHEMICAL AND FOOD TECHNOLOGY

Reg. No.: FCHPT-19990-50920

MPC-based Reference Governors: Theory and Applications

DISSERTATION THESIS

Study programme:	Process Control
Study field:	5.2.14. Automation
Department:	Institute of Information Engineering, Automation, and Mathematics
Thesis supervisor:	doc. Ing. Michal Kvasnica, PhD.

2017 **Ing. Martin Klaučo**

Slovak University of Technology in Bratislava
Faculty of Chemical and Food Technology
Institute of Information Engineering, Automation

DISSERTATION THESIS TOPIC

Author of the thesis:	Ing. Martin Klaučo
Study programme:	Process Control
Study field:	5.2.14. automation
Registration number:	FCHPT-19990-50920
Student's ID: :	50920
Thesis supervisor:	doc. Ing. Michal Kvasnica, PhD.
Title of the thesis:	MPC-based Reference Governors: Theory and Applications
Language of thesis:	English
Date of entry:	02. 09. 2013
Date of submission:	01. 06. 2017

Ing. Martin Klaučo
Solver

prof. Ing. Miroslav Fikar, DrSc.
Head of department

prof. Ing. Miroslav Fikar, DrSc.
Study programme supervisor

To my grandparents, Ol'ga and Juraj

Acknowledgments

First and foremost, I would like to express my sincere gratitude to my PhD supervisor, Michal Kvasnica. I received a lot of help, guidance, and inspiration, especially in the beginning of my PhD studies at the Slovak University of Technology in Bratislava. I would also like to thank Miroslav Fikar, the head of the department for creating a suitable and friendly work environment. I wish to mention Boris Rohál-Il'kiv, from Faculty of Mechanical Engineering, and Danica Rosinová, from the Faculty of Electrical Engineering for the insightful conversations during our joint seminars. On top of that, my thanks also goes to Francesco Borrelli, from UC Berkeley, where I briefly participated on research as visiting scholar.

This dissertation thesis would not be of such a quality without the help on joint research with my fellow PhD students and colleagues, namely Juraj Holaza, Ján Drgoňa, Martin Kalúz and Juraj Oravec.

Last, but not least, the biggest thank goes to my parents and grandparents for steering me towards academic work and for their support throughout this PhD journey.

Abstract

This dissertation thesis deals with the improvement of the control performance of closed-loop systems via optimizing the setpoints for these closed-loops. Closed-loops consist of a primary controller, which provides stability and some tracking properties for a process. Since these controllers are usually of very simple structure, they do not provide constraint satisfaction nor enforce tracking properties. This thesis summarizes a concept of reference governors (RG) based on model predictive control (MPC), which provides optimal setpoints for the primary control layer. We will show how such an MPC-based reference governors (MPC-RG) are formulated. We will be dealing with three main cases, first, a closed-loop containing PID controllers, second a closed-loop utilizing the behavior of the on/off controller and finally a complex closed-loop scheme with local MPC controllers. We will show how to model these closed-loops and subsequently formulate the MPC-RG control problems, which are in the form of optimization problems. The second part of this thesis is devoted to case studies, where the benefits of the MPC-RG strategy is elaborated. We consider one experimental case study involving the stabilizing and control of a magnetically suspended ball. Next, we offer two simulation-based case studies, one focused on improving the behavior of a boiler-turbine system and the second concerning a thermostatically controlled temperature in buildings.

Abstrakt

Táto dizertačná práca sa venuje návrhu riadiacich systémov, ktoré vylepšujú správanie sa už existujúcich riadiacich slučiek. Tieto existujúce riadiace slučky pozostávajú z primárneho regulátora, ktorý zabezpečuje stabilitu riadeného procesu a základné kvalitatívne parametre riadenia. Vylepšenie je realizované na základe optimalizačných metód, ktoré upravujú žiadané hodnoty existujúcich riadiacich slučiek. Keďže tieto primárne regulátory majú v drvivej väčšine jednoduchú štruktúru, nie je možné pomocou nich zabezpečiť prevádzku zariadenia, ktorá by spĺňala technologické normy a zároveň by bolo zabezpečené znižovanie finančných nárokov a pod. V tejto práci sa budeme zaoberať návrhom prediktívneho riadenia (model predictive control – MPC, z ang.), kde tento MPC regulátor bude poskytovať optimálne nastavenie žiadanej hodnoty pre primárnu vrstvu riadenia, t.j. bude z neho MPC supervízor. V tejto práci ukážeme ako navrhnúť takýto MPC regulátor pre tri hlavné triedy primárnych riadiacich slučiek. V prvej časti sa budeme venovať modelovanie a návrhu MPC pre slučky s PID regulátormi. Následne prejdeme na modelovanie a návrh MPC regulátore pre slučky s logickým riadením, alebo on/off regulármi. Ako posledný scenár budeme uvažovať, že primárna riadiaca slučka už obsahuje MPC riadiacu stratégiu, ale je potrebné ju vylepšiť. V druhej časti tejto dizertačnej práci ukážeme tri prípadové štúdie, kde demonštrujeme výsledky použitia MPC supervízorov. Prvá štúdia zahŕňa výsledky získane z experimentov pri riadení polohy guličky v magnetickom poli. Ďalej sa ukážeme výhody MPC supervízorov pri riadení energeticky náročných systémov ako je turbína spojená s výparným kotlom. Posledná prípadová štúdia zahŕňa návrh MPC supervízora pre termostatom riadenú teplotu v budovách.

Contents

CHAPTER 1

Introduction

Historically, in the process industry, the optimization based control was always used as a supervisory decision maker. More specifically, the optimization based algorithms just provided setpoints for primary layer of controllers, which were primarily responsible for the operation of processes. Such elaborated control schemes were firstly implemented in early 80's in the petrochemical industry. One of the most cited works in the history of the model prediction control is the paper by Cutler and Ramaker [1979], where the MPC predecessor, the dynamic matrix control (DMC) was introduced. This was one of the first scientific paper, which introduced a truly optimal solution to the optimization problem stemming from the industry needs. Here, the DMC controller was responsible for providing the setpoints for primary level controllers, like PIDs etc. The concept, in fact, was not new at all. A paper by Richalet et al. [1978] predates the DMC paper, but the solution to the optimization layer was done in a heuristic way. Such algorithms proved to be very effective in reducing the operating costs as well as increasing the safety of the entire plant operation [Qin and Badgwell, 2003]. Such a level of economic and safe operation is difficult to achieve using traditional control loops, which typically involve PI/PIDcontrollers [Åström and Hägglund, 2006]. Therefore optimization-based control strategies, such as those based on Model Predictive Control (MPC) [Maciejowski, 2002], are preferred in many areas, such as in petrochemical industries.

1.1 Concept of the MPC-based Governors

The concept, where the model predictive control strategy supplies setpoints for primary controller is referred to as an optimization-based reference governors. Such an arrangement of control layers was studies widely by Bemporad and Mosca [1994], Mosca [1996], Gilbert and Kolmanovsky [1999]. Results in these publication were oriented towards industrial applications. However,

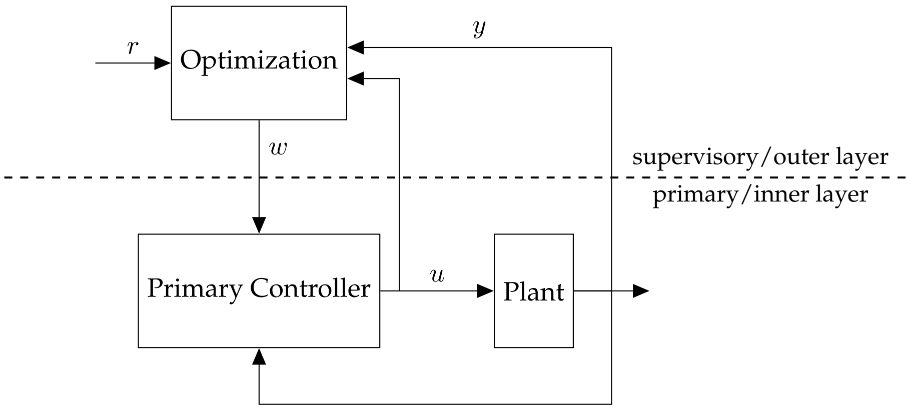


Figure 1.1: Conceptual diagram of the optimization-based reference governor scheme.

by optimizing the reference signals, the overall performance of the plant is improved [Bemporad, 1998, Gilbert and Kolmanovsky, 2002].

In this thesis, we follow more recent work by e.g. Borrelli et al. [2009], and we formulate and solve the reference governor problems as an optimal control problems. This, combined with utilization of the off-the-shelve optimization solvers yields better performance and widens the applicability of the governors.

A conceptual diagram is drawn in Fig. 1.1 to better illustrate the concept of the reference governor. Here, optimization-based layer is denoted as secondary layer or outer layer. Primary controllers are also referred to as inner controllers, which actually provides the control action u. Moreover, the user defined reference, denoted as r enters the secondary layer and the setpoint for the primary controller is denoted as w and it is obtained by solving the optimization problem in the upper layer, with respect to the measurements y. In this arrangements, the optimization variables is no longer the control input u, like it is in the situation where the MPC itself provides the value of the manipulated variable.

1.2 Contributions of the Thesis & Thesis Concept

1.2.1 Goals of the Thesis

Aims of the thesis were summarized into three main items, specifically

I. development of computationally efficient optimal control algorithms,

II. verification and implementation of optimal control algorithms on laboratory devices,

III. application of optimization and optimal control approached to path planning problems.

This thesis further explores the first two items. An extensive part of the theoretical contribution of the thesis is devoted to the closed-loop modeling, which is a necessary step before any control design. The main contribution in the area of the design of the control algorithms lies in structuring the MPC-based reference governors for three main cases of the closed-loop systems. Each of the proposed optimal control algorithms has been verified via simulation and experimental case studies. The most important results of the author in the area of optimal algorithm design related to the MPC-based reference governors were published in:

- Klaučo, M., Kvasnica, M.: Control of a boiler-turbine unit using MPC-based reference governors. *Applied Thermal Engineering*, Elsevier, ISSN: 1359-4311, vol. 110, pg. 1437–1447, 2017, (IF: 3.043), citations: 1.

- Klaučo, M., Kalúz, M., Kvasnica, M.: Real-time implementation of an explicit MPC-based reference governor for control of a magnetic levitation system. *Control Engineering Practice*, Elsevier, ISSN: 0967-0661, vol. 60, pg. 99–105, 2017. (IF: 1.830)

- Drgoňa, J., Klaučo, M., Kvasnica, M.: MPC-Based Reference Governors for Thermostatically Controlled Residential Buildings. In 54rd IEEE Conference on Decision and Control, Osaka, Japan, vol. 54, 2015,

- Holaza, J., Klaučo, M., and Kvasnica, M.: Solution techniques for multi-layer MPC-based control strategies (*accepted*). In Preprints of the 20th IFAC World Congress, France, pages –, 2017.

The author has also participated on research covering other areas of the control design, however, the results of that research are not elaborated in this thesis. Specifically, advances in the area of explicit MPC has been made by exploring the applicability of the region-less explicit MPC strategy. Results were published in

- Drgoňa, J., Klaučo, M., Janeček, F., Kvasnica, M.: Optimal control of a laboratory binary distillation column via regionless explicit MPC. *Computers &*

Chemical Engineering, Elsevier, ISSN: 0098-1354, vol. 96, pg. 139–148, 2017, (IF: 2.581).

Furthermore, the author made contributions in the optimal path-planning design:

- Oravec J., Klaučo, M., Kvasnica, M., Lofberg, J., Computationally Tractable Formulations for optimal Path Planning with Interception of Targets' Neighborhoods, *Journal of Guidance, Control, and Dynamics*, issue 5, vol. 40, pp. 1221–1230, 2017, American Institute of Aeronautics and Astronautics, ISSN: 0731-5090, (IF: 1.291)
- Klaučo, M., Blažek, S., Kvasnica, M.: An Optimal Path Planning Problem for Heterogeneous Multi-Vehicle Systems. *International Journal of Applied Mathematics and Computer Science*, ISSN 2083-8492, issue. 2, vol. 26, pg. 297–308, 2016. (IF: 1.037), citations: 1.

Full publication list of the author can be found in the Appendix A, which also includes conference publications related to the topics of this thesis as well other research areas.

1.2.2 Thesis Outline

This thesis is structured into two main parts. The first part is devoted to the theoretical contributions and the second part discusses experimental results and simulation-based case studies. In the theoretical part, we start with the preliminaries. The general optimization is introduced in the Chapter 2, which is followed by the introduction of model predictive control in the Chapter 3. Next, we will discuss the reference governors based on model predictive control. The Chapter 4 establishes the problem statement of the MPC-based reference governor. Moreover, this chapter includes three main sections, where particular MPC-RG control problem formulations are addressed. In the Section 4.1, we discuss closed-loop system involving a set of PID controllers. The Section 4.2 focuses on a closed-loop model involving the on/off controller. Finally, the Section 4.3 addresses the problem of MPC-RG for a case of a closed-loop model involving several local MPC strategies.

The second part of the thesis discusses the applications of proposed theoretical MPC-RG formulations. The first case study, in the Chapter 5 introduces experimental results involving the process of a magnetically suspended ball, where the MPC-RG is implemented as an explicit MPC controller. Next, in the

Chapter 6 we focus on a energy-intensive system, the boiler-turbine unit, where we show how the MPC-RG improves the behavior of three decoupled PI loops stabilizing the plant. Finally, the Chapter 7 presents a simulation-based case study involving optimizing the behavior of thermostatically controlled temperature inside a building. Conclusions and future research topics are addressed in the Chapter 8.

Part I

Theory

CHAPTER 2

Optimization

An overview of optimization is addressed in this chapter. In the Section 2.1, we introduce three key types of optimization problems. We cover the standard formulation of linear and quadratic programming problems. The MPC formulation and as well as MPC-based reference governors must be transformed into these standard forms of linear or quadratic problems so that they can be solved via available techniques. We also cover a mixed-integer optimization problem, which arises when considering MPC-based governors supervising relay-based closed-loops. Next, in the Section 2.2 we discuss solution techniques to obtain the optimal solutions to aforementioned optimization problems.

2.1 Types of Optimization Problems

A general optimization problem is stated as

$$\min_{z} \; h_{\text{obj}}(z) \tag{2.1a}$$

$$\text{s.t.} \; h_{\text{ineq},i}(z) \leq 0, \qquad \forall i \in \mathbb{N}_1^{m_{\text{ineq}}}, \tag{2.1b}$$

$$h_{\text{eq},j}(z) = 0, \qquad \forall j \in \mathbb{N}_1^{m_{\text{eq}}}, \tag{2.1c}$$

where the vector $z \in \mathbb{R}^m$ denotes the optimization variable, $h_{\text{obj}}(z)$ is the objective function, and $h_{\text{ineq},i}(z),\ h_{\text{eq},j}(z)$ are constraint functions. Scalars m_{ineq}, and m_{eq} represents the number of inequality constraints and equality constraints, respectively. Based on the type of the objective function (2.1a) and functions $h_{\text{ineq},i}(z),\ h_{\text{eq},j}(z)$, we distinguish between several classes of the optimization problems.

Firstly, we differentiate between convex optimization problems and non-convex. The main advantage of convex optimization problems over the non-convex problems is the guaranteed presence of unique global optima $z^{\star}$. Problems are convex, if and only if the objective function and all constraint functions are convex. Particular optimization problems are discussed in subsequent sections. For more details about any of mentioned class of optimization problems, we direct the reader to [Boyd and Vandenberghe, 2009, Nocedal and Wright, 2006].

2.1.1 Linear Programming

When the objective function, as well as the constraint functions, are affine, we refer to such an optimization problem as a linear optimization problem (LP). Linear programming problems are usually written in the matrix form

$$\min_{z} \quad c^\intercal z + d \tag{2.2a}$$

$$\text{s.t.} \quad A_{\text{ineq}} z \leq b_{\text{ineq}}, \tag{2.2b}$$

$$A_{\text{eq}} z = b_{\text{eq}}, \tag{2.2c}$$

where $z \in \mathbb{R}^m$ is the vector of optimization variables. Next, $c \in \mathbb{R}^m$, d is a scalar, $A_{\text{ineq}} \in \mathbb{R}^{m_{\text{ineq}} \times m}$, $b_{\text{ineq}} \in \mathbb{R}^{m_{\text{ineq}}}$, $A_{\text{eq}} \in \mathbb{R}^{m_{\text{eq}} \times m}$, and $b_{\text{eq}} \in \mathbb{R}^{m_{\text{ineq}}}$ are constants of the LP. The LP is a convex optimization problem.

2.1.2 Quadratic Programming

The quadratic optimization problem (QP) differs from the LP in the objective function. Specifically, we cast the quadratic optimization problems as follows

$$\min_{z} \quad z^\intercal H z + c^\intercal z + d \tag{2.3a}$$

$$\text{s.t.} \quad A_{\text{ineq}} z \leq b_{\text{ineq}}, \tag{2.3b}$$

$$A_{\text{eq}} z = b_{\text{eq}}, \tag{2.3c}$$

with the vector $z \in \mathbb{R}^m$ being the decision variable. The matrix $H \in \mathbb{R}^{n \times n}$ determines the convexity of the optimization problem. If $H \succ 0$, then the optimization problem in (2.3) is a convex problem, hence it solution converges to a unique optimum.

2.1.3 Mixed-Integer Programming

Presented formulations in Section 2.1.1 and 2.1.2 assume that the optimization variable z is real. Here, we split the optimization variable z to a vector of real variables and to a vector of binary variables, specifically,

$$z = \begin{bmatrix} z_{\text{r}}^\intercal & z_{\text{b}}^\intercal \end{bmatrix}^\intercal, \tag{2.4}$$

where $z_\mathrm{r} \in \mathbb{R}^{m_\mathrm{r}}$, $z_\mathrm{b} \in \{0,1\}^{m_\mathrm{b}}$, and $m_\mathrm{r} + m_\mathrm{b} = m$. In this thesis, we will deal with mixed-integer (MI) optimization problems with linear constraints and with linear (MILP) or quadratic (MIQP) objective function. Formally, these MI problems can be stated as

$$\min_{z_\mathrm{r}, z_\mathrm{b}} \; h_\mathrm{obj}(z_\mathrm{r}, z_\mathrm{b}) \tag{2.5a}$$
$$\text{s.t.} \; A_\mathrm{ineq,r} z_\mathrm{r} + A_\mathrm{ineq,b} z_\mathrm{b} \leq b_\mathrm{ineq}, \tag{2.5b}$$
$$A_\mathrm{eq,r} z_\mathrm{r} + A_\mathrm{eq,b} z_\mathrm{b} = b_\mathrm{eq}, \tag{2.5c}$$
$$z_\mathrm{b} \in \{0,1\}^{m_\mathrm{b}} \tag{2.5d}$$

with $A_\mathrm{ineq,r}$, $A_\mathrm{ineq,b}$, $A_\mathrm{eq,r}$, and $A_\mathrm{eq,b}$ of real matrices of appropriate size. If the optimization problem (2.5) is casted as an MILP, then the objective function is

$$h_\mathrm{obj}(z_\mathrm{r}, z_\mathrm{b}) = c_\mathrm{r}^\intercal z_\mathrm{r} + c_\mathrm{b}^\intercal z_\mathrm{b} + d, \tag{2.6}$$

while for an MIQP,

$$h_\mathrm{obj}(z_\mathrm{r}, z_\mathrm{b}) = z_\mathrm{r}^\intercal H_\mathrm{r} z_\mathrm{r} + z_\mathrm{b}^\intercal H_\mathrm{b} z_\mathrm{b} + c_\mathrm{r}^\intercal z_\mathrm{r} + c_\mathrm{b}^\intercal z_\mathrm{b} + d. \tag{2.7}$$

By including the binary constraint (2.5d), the problem (2.5) becomes non-convex, which makes it more difficult to solve then LP/QP. Further reading material on the topic of the mixed-integer programming can be found in [Conforti et al., 2014].

2.2 Solutions Techniques

In this section we briefly discuss the solutions techniques to aforementioned optimization problems. We will focus on the traditional optimization techniques, denoted as online optimization and we will elaborate on the parametric optimization as well. We will not cover the method in detail, since it is not the topic of this thesis.

2.2.1 Online Optimization

By the term online optimization we mean that the optimization problem is numerically solved, hence obtaining the optimal value of decision variables. The optimization problems are solved using one of many algorithms which largely depends on formulating the KKT conditions.

The best known algorithms for obtaining the optimal solution to QP and LP is the *Active-Set Method* and the *Interior-Point Method*. For the LP we can employ the

Simplex Method to find the optimum. Solving this class of optimization problems is fairly easy, and it can be done even for a large number of decision variables in reasonable time, by employing state-of-the-art solvers, like GUROBI, CPLEX, and MOSEK. On the other hand, solving the MILP/MIQP is more time consuming. The brute-force way to solve MI problems is to enumerate all feasible combinations of the binary variable and then solve corresponding LP/QP via, e.g., Active-Set. Then, the optimal combination of binary variable will be chosen such that the value of the objective function is as small as possible. Since there are n_b variables, the worst case scenario is to solve 2^{n_b} local LPs/QPs. However, more effective ways exist to solve the MILP and MIQP problems. Software tools like GUROBI[1], CPLEX[2], and MOSEK[3] utilize methods like *Branch&Bound* and *Branch&Cut* algorithms, which allow for faster solution of the MI problem without the need of exploring all of the combinations of binary variable.

2.2.2 Parametric Optimization

We will discuss the principle of the parametric optimization on the quadratic programming (2.3). Consider a QP[4] into

$$\min_z \ z^\intercal H z + \theta^\intercal F z + c^\intercal z \tag{2.8a}$$

$$\text{s.t. } A_\mathrm{ineq} z \leq b_\mathrm{ineq} + E\theta, \tag{2.8b}$$

where $z \in \mathbb{R}^m$ is the vector of decision variables and $\theta \in \mathbb{R}^{n_\theta}$ is a free parameter. In connection with the MPC, the θ parameter is usually an initial condition or a measurement of a process variable. Next, H, F, c, A_ineq, b_ineq and E are matrices and vectors of constants of appropriate sizes. Naturally, if the θ would be given, the problem (2.8) translates back to the QP form in (2.3). In the parametric optimization, however, the parameter is not known a priori. Here, the solution to the problem is not a specific value $z^\star$, but an explicit function of all possible values of θ, which satisfy the constraint (2.8b). The optimal solution si denoted as $z^\star(\theta)$. Once the $z^\star(\theta)$ is constructed, the optimal value of the decision variable is obtained by a mere function evaluation at the given value of θ variable. Note, that parametric solution and subsequent evaluation of $z^\star(\theta)$ for given θ is equivalent to the online solution. The parametric formulation of the optimization problem can be extended to accommodate the quadratic objective function as well as mixed-integer cases. Depending on the structure

[1] www.gurobi.com

[2] www-01.ibm.com/software/commerce/optimization/cplex-optimizer/

[3] www.mosek.com

[4] The equality constraints can be removed from the optimization problem by taking the nullspace of the matrix A_eq and some particular solution for z in (2.2c)

of the original problem, the properties of the resulting $z^\star(\theta)$ changes. In case of LP and QP, the $z^\star(\theta)$ is in the form of a continuous piecewise affine (PWA) function, while in case of the MILP and MIQP the resulting optimizer can be discontinuous PWA function. Specifically,

LEMMA 2.1 (BEMPORAD ET AL. [2002], BORRELLI [2003]). Consider the parametric quadratic program in (2.8). Then the optimizer $z^\star = \kappa(\theta)$ is a piecewise affine (PWA) function of the vector of parameters, i.e.,

$$\kappa(\theta) = \begin{cases} \alpha_1\theta + \beta_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ \alpha_{n_\mathrm{R}}\theta + \beta_{n_\mathrm{R}} & \text{if } \theta \in \mathcal{R}_{n_\mathrm{R}} \end{cases}. \tag{2.9}$$

where

$$\mathcal{R}_i = \{\theta \mid \Gamma_i\theta \leq \gamma_i\} \quad i = 1, \ldots, n_\mathrm{R} \tag{2.10}$$

are polyhedral regions and n_R denotes the total number of regions. □

In simple terms, Lemma 2.1 states that the optimal solution to (2.8) for a particular value of the parameter θ can be obtained by simply evaluating the PWA function in (2.9). This can be done in various ways. The simplest one is to use the so-called *sequential search*. Here, we go through each $i = 1, \ldots, n_\mathrm{R}$, checking whether $\theta \in \mathcal{R}_i$. This can be done by testing whether $\Gamma_i\theta \leq \gamma_i$ holds. If θ is contained in $\mathcal{R}_i$, the optimal value of $z^\star$ is given by $z^\star = \alpha_i\theta + \beta_i$ and the procedure can be aborted. Otherwise the counter i is incremented by $i = i + 1$ and the next region is tested. Trivially, the runtime of such a sequential procedure is $\mathcal{O}(n_\mathrm{R})$. More efficient evaluation algorithms exist, e.g. the binary search tree procedure of Tøndel et al. [2003], which is schematically depicted in the Fig. 2.1. These, however, require additional preprocessing effort. Finally, we remark that the parametric representation of the optimizer in (2.9) can be obtained off-line by applying parametric programming solvers [Oberdieck et al., 2016a], which are available, e.g., in the Multi-Parametric Toolbox [Herceg et al., 2013], or in the POP toolbox [Oberdieck et al., 2016b].

The practical implication of the parametric solution is the ability to use techniques of the optimal control in places, where online-solvers can not be implemented. We may mention a low-level hardware like micro-controllers, which are unable to accommodate complex algorithms like Active-Set [Kalúz et al., 2015]. There are however several disadvantages to the parametric programming, mainly the complexity of the obtain the parametric solution or the memory demands for storing the optimizer $z^\star(\theta)$. Further reading material on this topic can be found in [Borrelli et al., 2015, Kvasnica, 2009, Bemporad et al., 2002].

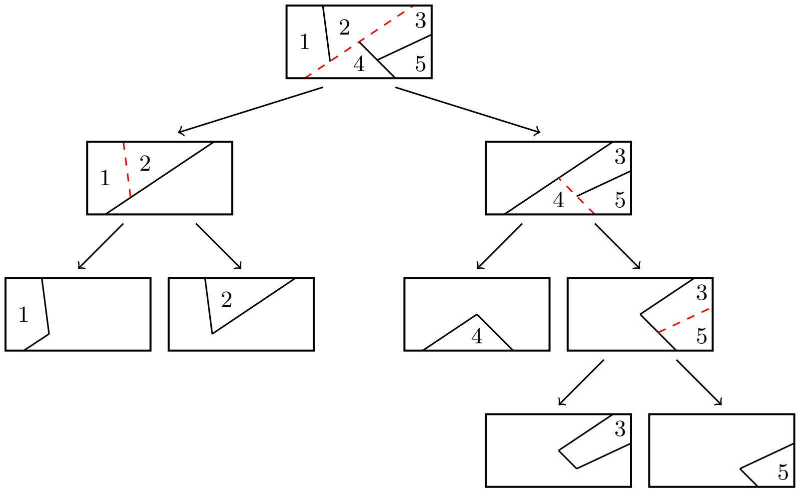


Figure 2.1: Binary search tree.

2.3 Concluding Remarks

This chapter was devoted to the presentation of the optimization problems. We have presented linear programming, quadratic programming, and mixed-integer programming. These problems are directly linked to the model predictive control strategy, which is shown in the next chapter. The MPC is formulated as an optimal control problem, which is then translated into an LP, QP, MILP or MIQP problem, to be solved. In this thesis, we use both approaches to solving MPC problems. Further details about particular applications of the optimization can be found in case studies which are discussed in Part II of this thesis.

CHAPTER 3

Model Predictive Control

The main principle of the model predictive control strategy is to choose optimal control inputs while predicting the future behavior of the plant based on the current measurements of process variables. The future behavior is predicted over a time frame called the prediction horizon. Repeated solving an optimization problem obtains the optimal control actions at every time instant. To such a scheme we often refer to as a receding horizon policy [Maciejowski, 2002, Mayne et al., 2000]. Naturally, the topic of the MPC is widely studied by many researchers in connection with various fields [Mayne, 2014]. Theory presented in this chapter of the thesis is inspired mainly by work of [Pannocchia and Rawlings, 2003, Shead et al., 2009, Muske and Badgwell, 2002] in terms of the MPC formulations and especially in connection with offset free control strategies.

These optimization problems are of various nature, depending on the type of the plant we desire to control or depending on the hardware where the control actions are evaluated. In this chapter, we will present the control problem formulations if we assume that the plant is controlled solely by the MPC. To such a setup we will refer to as a Direct-MPC scheme. In every case of the MPC design, a knowledge of the plant model is required. Naturally, we know of several types of model, which can be used in the synthesis of the optimization-based controllers.

Despite the choice of the plants' model, the structure of the MPC controller remains the same, in general, we can cast it as follows

$$
\begin{aligned}
\min_{U} \quad & \ell_{\mathrm{N}}(x_N) + \sum_{k=0}^{N-1} \ell\left(x_k, y_k, u_k\right) && && \text{(3.1a)}\\
\text{s.t.} \quad & x_{k+1} = f(x_k, u_k), && k \in \mathbb{N}_0^{N-1} && \text{(3.1b)}\\
& y_k = g(x_k, u_k), && k \in \mathbb{N}_0^{N-1} && \text{(3.1c)}\\
& u_k \in \mathcal{U}, && k \in \mathbb{N}_0^{N-1} && \text{(3.1d)}\\
& x_k \in \mathcal{X}, && k \in \mathbb{N}_0^{N-1} && \text{(3.1e)}\\
& y_k \in \mathcal{Y}, && k \in \mathbb{N}_0^{N-1} && \text{(3.1f)}\\
& x_0 = x(t). && && \text{(3.1g)}
\end{aligned}
$$

Here, constraints (3.1b) and (3.1c) represent the model behavior based on which the predictions are made. Next, we enforce limitations on manipulated, state and process variables via (3.1d)-(3.1f). Sets $\mathcal{U}$, $\mathcal{X}$, and $\mathcal{Y}$ can be of arbitrary nature, but we usually restrict signals u, x and y by a box constraint, expressed as

$$
\begin{aligned}
u_{\min} \leq u_k \leq u_{\max}, && \text{(3.2a)}\\
x_{\min} \leq x_k \leq x_{\max}, && \text{(3.2b)}\\
y_{\min} \leq y_k \leq y_{\max}. && \text{(3.2c)}
\end{aligned}
$$

The quality criteria of the control performance are utilized in the objective function (3.1a). The optimization problem (3.1) is initialized by a measurement of the state variables $x(t)$ in (3.1g). The vector of initial conditions may differ from particular choice of the prediction model in (3.1b) or by choice of the objective function. The solution of the aforementioned optimization problem yields an optimal sequence of control inputs

$$
U = \begin{bmatrix} u_0^{\intercal} & \dots & u_{N-1}^{\intercal} \end{bmatrix}^{\intercal}. \tag{3.3}
$$

In the closed-loop implementation, we apply to the process the first element of the sequence u_0 and we disregard the rest of the sequence $\begin{bmatrix} u_1^{\intercal} & \dots & u_{N-1}^{\intercal} \end{bmatrix}^{\intercal}$. In the next sampling instant, we resolve the (3.1) with a new initial condition x_0. Such an implementation is called Receding Horizon Control (RHC).

Depending of the choice of the objective function (3.1a) and the choice of the functions $f(\cdot)$, $g(\cdot)$ we may arrive to various types of optimization problems. First, we will focus on the types of the objective function. Terms of the objective function are expressed via weighted norm $\|Qz\|_p$, where $p = \{1, 2, \infty\}$, $Q \succ 0$, and $z \in \mathbb{R}^m$. The first term of the objective function, the terminal penalty, is an optional extension of the performance criteria. The choice of the value p then translates the weighted norm to specific expressions

$$\|Qz\|_1 = \sum_{i=1}^{m} |Q_i z_i|, \tag{3.4a}$$

$$\|Qz\|_2 = z^\mathsf{T} Q z, \tag{3.4b}$$

$$\|Qz\|_\infty = \max\left(|Q_i z_i|\right). \tag{3.4c}$$

From (3.4) results that if we choose $p = \{1, \infty\}$ the resulting objective function will be linear in the optimization variables, and if $p = 2$ we will arrive at quadratic objective function.

In this thesis we will deal with *tracking* types of the MPC, hence we will consider following structure of the objective function (3.1a)

$$\ell(y_k, u_k) = \|Q_\mathrm{y}\left(r_k - y_k\right)\|_p + \|Q_\mathrm{du} \Delta u_k\|_p, \tag{3.5}$$

where $\Delta u_k = u_k - u_{k-1}$. The first term drives the output y_k to the $k-$th step of the user defined reference r_k. The second term penalizes the control increments, which introduces integral action to the closed-loop. Such property is desired when the steady state offset needs to be removed.

The prediction model changes the structure of the constraints. In general, we can either choose a nonlinear expressions for the model behavior, or we consider a linear models. The first case results in a non-linear optimization problem, while the latter choice results in quadratic programming or linear programming.

Lastly, sets $\mathcal{U}$, $\mathcal{X}$, $\mathcal{Y}$ are usually chosen as polyhedra. Thus they are represented by finitely many linear inequalities. However, these set can also include non-linear expressions, like an ellipsoidal structure or even non-convex shapes. It is often desired to structure the MPC towards an LP or a QP with convex linear constraints. Such classes of optimization problem can be easily solved via several state-of-the-art tools.

3.1 Linear Prediction Models

This section presents standard prediction models. First, we will show the structure of the MPC if an input-output model in the form of a transfer function is used. Second, we focus on a state space prediction model. Both of these formulations are later utilized to design the MPC-based reference governor.

3.1.1 Input-output Prediction Model

The simplest choice for the plant model can be a transfer function. Such a model is usually used when we are dealing with SISO systems. In order to combine the transfer function based model

$$H(z^{-1}) = \frac{\sum_{j=0}^{m} b_j z^{-j}}{\sum_{i=0}^{n+1} a_i z^{-i}}, \tag{3.6}$$

with the MPC structure in (3.1) we require that the prediction model is in the form of the difference equation similar to (3.1b) or (3.1c). Naturally, if the transfer function is proper, i.e., $m \leq n$, then the model in (3.6) can be converted into the time domain, yielding

$$y(t+T_\mathrm{s}) = \frac{1}{a_0}\left(-\sum_{i=1}^{n} a_i y\big(t-(i+1)T_\mathrm{s}\big) + \sum_{j=0}^{m} b_j u\big(t-(j+1)T_\mathrm{s}\big)\right). \tag{3.7}$$

To simplify the notation, lets abbreviate the right hand side of (3.7) by

$$y(t+T_\mathrm{s}) = f_\mathrm{y}(y(t),\dots,y(t-nT_\mathrm{s}),u(t),\dots,u(t-mT_\mathrm{s})). \tag{3.8}$$

In order to use the difference equation in (3.8) as a prediction equation, we shift the sequence by a one sample time instant forward. The MPC problem formulation is then given as

$$\min_{u_1,\dots,u_N} \quad \ell_\mathrm{N}(y_{N+1}) \sum_{i=0}^{N-1} \ell_\mathrm{io}\left(y_k, u_k\right) \tag{3.9a}$$

$$y_{k+1} = f_\mathrm{y}(y_k,\dots,y_{k-n},u_k,\dots,u_{k-m}), \quad k \in \mathbb{N}_0^{N-1}, \tag{3.9b}$$

$$u_k \in \mathcal{U}, \quad k \in \mathbb{N}_0^{N-1}, \tag{3.9c}$$

$$y_k \in \mathcal{Y}, \quad k \in \mathbb{N}_0^{N-1}, \tag{3.9d}$$

$$\theta_0 = \theta(t), \tag{3.9e}$$

where the function $f_\mathrm{y}(\cdot)$ in (3.9b) is the prediction equation based on (3.8). The problem (3.9) is initialized by vector of theta given as

$$\theta(t) = \begin{bmatrix} y(t) & \dots & y(t-nT_\mathrm{s}) & u(t-T_\mathrm{s}) & \dots & u(t-mT_\mathrm{s}) \end{bmatrix}^\intercal. \tag{3.10}$$

Since the one-step-ahead prediction of the output $y(t+T_\mathrm{s})$ depends on the sequence of the current and past inputs and measured outputs respectively, the vector of initial conditions contains previous instances of y and u in accordance with (3.7).

The objective function (3.9) given as

$$\ell_{\text{io}}(y_k, u_k, r_k) = \|Q_{\text{ry}}\,(r_k - y_k)\,\|_2 + \|Q_{\text{u}}\,(\Delta u_k)\,\|_2, \tag{3.11}$$

consists of two terms, the first penalizes the control error in each step of the prediction and the latter one penalizes the differences in control actions $\Delta u_k = u_k - u_{k-1}$. The vector $\theta(t)$ also consist of the term $u(t - T_{\text{s}})$, which is required for the initialization of the Δu_1 in (3.11).

Even though the model predictive control algorithms originated in the dynamic matrix control (DMC) Cutler and Ramaker [1979], later known as Generalized Predictive Control (GPC), which was based on the input-output models Camacho and Bordons [2007], the formulation (3.9) has several key limitations. First, we do not have access to the states, hence we also can not include constraints on these signals. Second, the vector of initial parameters $\theta(t)$ can be of significant length for higher order systems. This however has no direct consequence if we consider implicit solution to the optimization problem. On the other hand the length of θ has negative impact on the complexity of the explicit solutions to the QPs. The explicit model predictive control strategy is covered in more detail in the Section 3.3.

3.1.2 State Space Prediction Model

Second type of widely used MPC formulations stems from the utilization of state space models. Formally the state space model in discrete is expressed as

$$x(t + T_{\text{s}}) = Ax(t) + Bu(t), \tag{3.12a}$$

$$y(t) = Cx(t) + Du(t), \tag{3.12b}$$

where states are denoted as $x \in \mathbb{R}^{n_{\text{x}}}$, measured signals are denoted by $y \in \mathbb{R}^{n_{\text{y}}}$ and by $u \in \mathbb{R}^{n_{\text{u}}}$ is represented the vector of manipulated variables. Next, $A \in \mathbb{R}^{n_{\text{x}} \times n_{\text{x}}}$, $B \in \mathbb{R}^{n_{\text{x}} \times n_{\text{u}}}$, $C \in \mathbb{R}^{n_{\text{y}} \times n_{\text{x}}}$ and $D \in \mathbb{R}^{n_{\text{y}} \times n_{\text{u}}}$. The variable T_{s} represents the sampling time.

In fact, most of scientific works in the field of MPC strategies are based on linear state space models Prasath and Jørgensen [2009], Shead et al. [2010], Rawlings and Mayne [2009]. The control problem can be casted in several different ways. In this thesis we will focus on a formulation, which removes the offset, i.e. manages to track the reference. More on the offset-free reference tracking can be found in the next Section 3.2. The control problem in form of an optimization problem is expressed as follows

$$
\begin{aligned}
\min_{u_0,\ldots,u_{N-1}} \quad & \|Q_{\mathrm{N}} x_N\|_p + \sum_{k=0}^{N-1} \ell_{\mathrm{ss}}\left(x_k, y_k, u_k\right) && && \text{(3.13a)}\\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k, && k \in \mathbb{N}_0^{N-1} && \text{(3.13b)}\\
& y_k = C x_k + D u_k, && k \in \mathbb{N}_0^{N-1} && \text{(3.13c)}\\
& u_{\min} \leq u_k \leq u_{\max}, && k \in \mathbb{N}_0^{N-1} && \text{(3.13d)}\\
& x_{\min} \leq x_k \leq x_{\max}, && k \in \mathbb{N}_0^{N-1} && \text{(3.13e)}\\
& y_{\min} \leq y_k \leq y_{\max}, && k \in \mathbb{N}_0^{N-1} && \text{(3.13f)}\\
& x_0 = x(t), u_{-1} = u(t - T_{\mathrm{s}}). && && && \text{(3.13g)}
\end{aligned}
$$

where the objective function (3.13a) is given as

$$
\ell_{\mathrm{ss}}(\cdot) = \|Q_{\mathrm{ry}}\left(r_k - y_k\right)\|_2 + \|Q_{\mathrm{u}}\left(\Delta u_k\right)\|_2, \tag{3.14}
$$

in which again the last term is defined as $\Delta u_k = u_k - u_{k-1}$. If we assume that the design model given by (3.13b) and (3.13c), is identical to the process, then using such an MPC strategy will achieve offset-free tracking of given reference. Of course, this assumption is rarely valid, hence different techniques must be considered in order to make the MPC track the reference precisely. The next section will cover such a control strategy.

3.2 Offset-Free Control Scheme

As indicated in the previous section, we rarely arrive at the situation, when the design model is identical to the process. Moreover, even if this is the case, the process is usually affected by unmeasured disturbances, which introduces the offset to the tracking problem. In order to mitigate this situation, we extend the design model by a set of state and output disturbance signals, which are to be estimated at every sampling instant. This control scheme is often called as disturbance modeling, and it is well documented in several publications Muske and Rawlings [1993], Muske [1997], Muske and Badgwell [2002].

Since the offset-free control strategy plays a vital role in this thesis, lets establish the procedure. First, consider an extended state space model in discrete time

$$
\begin{aligned}
x_{k+1} &= A x_k + B u_k + E_{\mathrm{x}} d_k, && \text{(3.15a)}\\
y_k &= C x_k + D u_k + E_{\mathrm{y}} d_k, && \text{(3.15b)}\\
d_{k+1} &= d_k, && \text{(3.15c)}
\end{aligned}
$$

where the unmeasured disturbances d enter into (3.15a) and (3.15b) via a user-specified matrices of appropriate sizes E_x, E_y, respectively. Moreover, the disturbances are assumed to have constant dynamics, cf. (3.15c). Define the estimated extended state vector as

$$\hat{x}_\mathrm{e} = \begin{bmatrix} \hat{x}_k \\ \hat{d}_k \end{bmatrix}, \tag{3.16}$$

where $\hat{x}$ is the estimate of the state vector of the original model in (3.13), and $\hat{d}$ is the estimate of the unmeasured disturbances. The number of these disturbances can vary, depending on the controller process. Usually, as suggested by many scientific works, e.g. by [Borrelli et al., 2015, ch. 13.6] the number of disturbances should coincide with the number of controller outputs. The choice of matrices $E_\mathrm{x}, E_\mathrm{y}$ depends also on the controller process. Usually, we set the matrix E_x as a zero matrix if the process does not contain unstable open loop dynamics Pannocchia [2003].

Next, we have two choices of obtaining the estimate of the vector $x_{\mathrm{e},k}$. Either we use a standard Luenberger observer, or we consider a time-varying Kalmar Filter. The first choice, the Luenberger observer is implemented via a state space model

$$x_{\mathrm{e},k+1} = A_\mathrm{e} x_{\mathrm{e},k} + B_\mathrm{e} u_k + L\left(y_{\mathrm{e},k} - y_{\mathrm{m},k}\right), \tag{3.17a}$$
$$y_{\mathrm{e},k} = A_\mathrm{e} x_{\mathrm{e},k} + D_\mathrm{e} u_k, \tag{3.17b}$$

where the system matrices are defined as follows

$$A_\mathrm{e} = \begin{bmatrix} A & E_\mathrm{x} \\ 0 & I \end{bmatrix}, \qquad B_\mathrm{e} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \tag{3.18a}$$
$$C_\mathrm{e} = \begin{bmatrix} C & E_\mathrm{y} \end{bmatrix}, \qquad D_\mathrm{e} = \begin{bmatrix} D \end{bmatrix}, \tag{3.18b}$$

and the variable $y_\mathrm{m}(t)$ is the vector measurements taken from the process. The gain L is a static gain obtain usually via a pole placement method. In Pannocchia and Rawlings [2003] is also suggested that L can be calculated with respect to the stochastic properties of the state and output signals, i.e., to use a static Kalman Filter. The main advantage of using static gain in the observer is the simple structure of the estimator, hence easier implementation. Naturally, if such an approach of state estimation is used in connection with a non-linear model, the actual control performance can be negatively affected if we move away from the linearisation point, at which the model in (3.15) was obtained. In order to mitigate this negative impact, we propose to use a time-varying Kalman Filter, which has better convergence properties even if we move away from steady state.

The time-varying Kalman filter procedure consists of two phases. The first phase is the prediction phase which is followed by the update phase. In both phases the index $k|k$ represents current estimate of the individual variables. The prediction phase consist of two equations, namely

$$\hat{x}_{\mathrm{e},k|k-1} = A_\mathrm{e}\hat{x}_{\mathrm{e},k-1|k-1} + B_\mathrm{e}u_k, \tag{3.19a}$$

$$P_{k|k-1} = A_\mathrm{e}P_{k-1|k-1}A_\mathrm{e}^\intercal + Q_\mathrm{e} \tag{3.19b}$$

where $\hat{x}_{\mathrm{e},k|k-1}$ is the predicted state estimate based on the previous time instant, and $P_{k|k-1}$ is the predicted value of the covariance matrix. The matrices A_e, B_e, C_e, D_e are given as per (3.18).

The consecutive step in the estimation algorithm is the update phase, represented by

$$\epsilon_k = (y_{\mathrm{m},k} - y_\mathrm{L}) - \left(C_\mathrm{e}\hat{x}_{\mathrm{e},k|k-1} + D_\mathrm{e}w_k\right), \tag{3.20a}$$

$$S_k = C_\mathrm{e}P_{k|k-1}C_\mathrm{e}^\intercal + R_\mathrm{e}, \tag{3.20b}$$

$$L_k = P_{k|k-1}C_\mathrm{e}^\intercal S_k^{-1}, \tag{3.20c}$$

$$\hat{x}_{\mathrm{e},k|k} = \hat{x}_{\mathrm{e},k|k-1} + L_k\epsilon_k, \tag{3.20d}$$

$$P_{k|k} = (I - L_kC_\mathrm{e})\,P_{k|k-1}. \tag{3.20e}$$

Here, the estimation error ϵ_k is calculated based on the plant measurements $y_{\mathrm{m},k}$ per (3.20a). Since the estimator runs in deviation variables, the linearisation point y_L must be substracted from the measurement. The time-varying estimator gain L_k is then calculated by (3.20b) and (3.20c). This gain is subsequently used to obtain the current estimate of the state variables $\hat{x}_{\mathrm{e},k|k}$ via (3.20d). At the end of the update phase, the covariance matrix P is updated. The tuning matrix R_e should be chosen with respect to the stochastic properties of the output signals.

The estimates of $\hat{x}_\mathrm{e}$ and d can then be extracted from $\hat{x}_\mathrm{e}$ by

$$\hat{x}_k = M_\mathrm{x}\hat{x}_{\mathrm{e},k|k}, \quad \hat{d}_k = M_\mathrm{d}\hat{x}_{\mathrm{e},k|k}, \tag{3.21}$$

where

$$M_\mathrm{x} = \begin{bmatrix} I_{n_\mathrm{x}} & 0 \end{bmatrix}, \quad M_\mathrm{d} = \begin{bmatrix} 0 & I_{n_\mathrm{d}} \end{bmatrix}. \tag{3.22}$$

Here, n_x is the number of states of the closed-loop system in (3.12) and n_d is the number of disturbances.

The MPC design directly follows from the extended state space model (3.18), specifically

$$\min_{U} \; \ell_{\mathrm{N}}(x_N) + \sum_{k=0}^{N-1} \ell_{\mathrm{ss}}\left(x_k, y_k, u_k\right) \tag{3.23a}$$

$$\text{s.t.} \; x_{k+1} = Ax_k + Bu_k + Ed_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{3.23b}$$

$$y_k = Cx_k + Du_k + Fd_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{3.23c}$$

$$u_{\min} \leq u_k \leq u_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{3.23d}$$

$$x_{\min} \leq x_k \leq x_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{3.23e}$$

$$y_{\min} \leq y_k \leq y_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{3.23f}$$

$$x_0 = \hat{x}(t), u_{-1} = u(t - T_{\mathrm{s}}), d_0 = \hat{d}(t). \tag{3.23g}$$

The objective function ℓ_{ss} remains the same as in (3.14). In many situations we do not have access to the future value of the disturbance signal, hence in the MPC design we often substitute the term d_k for d_0 in (3.23b) and (3.23c).

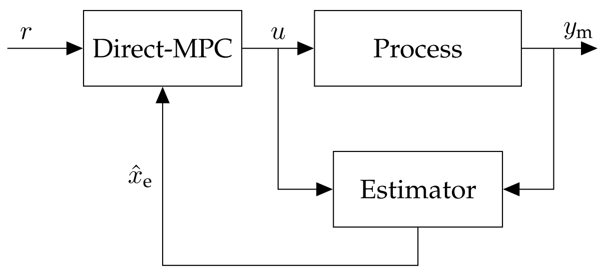


Figure 3.1: Control scheme of a MIMO PID control setup.

The whole procedure of the offset-free control scheme is visualized in the block diagram on the Fig. 3.1. The Direct-MPC block represents the optimization problem given in (3.23) and the Estimator block refers to either Luenberger observer or to the time-varying Kalman Filter.

3.3 Explicit MPC Concepts

Once the MPC problem is formulated as a quadratic optimization problem (QP), obtaining optimal control inputs amounts to solving the QP for the particular initial conditions. In essence there are two ways how to achieve this: the on-line (implicit) approach and the off-line (explicit) approach. In the on-line approach the QP is solved on-line at each sampling instant for a particular initial condition using numerical optimization methods. In the off-line approach the QP is solved for all feasibile initial conditions. By using parametric programming the optimal solution can be obtained as a function which maps initial conditions onto optimal control inputs. Then, on-line, the optimal control inputs are obtained by merely evaluating such a mapping function. It is worth emphasizing that both approaches yield the same optimal control moves and are thus, from a mathematical point of view, equivalent.

However, the off-line approach (see Section 2.2.2) offers one crucial advantage: the on-line evaluation of the optimal map is usually faster and simpler compared to solving the QP on-line. Therefore such an approach is ideal when aiming for a fast and simple implementation of the MPC strategy to devices such as PLCs.

The idea of explicit MPC as popularized mainly by Bemporad et al. [2002] is to use the parametric representation of the optimal solution to (2.8) to abolish the main limitation of traditional on-line methods in terms of long computation times. Specifically, in explicit MPC the idea is to replace on-line optimization by evaluation of the PWA solution in (2.9). If the number of regions, i.e., n_{R}, is low (say below 100), obtaining $U^{\star}(\theta)$ from (2.9) can be done in the range of microseconds even on a very simple implementation hardware. Recently, the topic of region-free explicit MPC has resurfaced [Kvasnica et al., 2015] originally suggested by Borrelli et al. [2010], hence the explicit MPC can be applied to process with larger number of parameters. Presented approach has been experimentally tested on the control of the reflux ratio in the distillation column and findings has published in [Drgoňa, Klaučo, Janeček, and Kvasnica, 2017].

An another, and probably even more prominent, advantage of the parametric solutions in (2.9) is that their implementation is *division free*. This means that identification of $U^{\star}(\theta)$ only requires additions and multiplications. No division operations are required. This mitigates potential implementation problems e.g. due to buffer overflow or division-by-zero scenarios.

Once optimal solution $U^{\star}(\theta)$ via explicit MPC is obtained, to the process is applied only the first component of $U^{\star}(\theta)$, usually denoted as $u^{\star}$. This follows the receding horizon policy.

As s final remark, it should be mention that the parameter θ does not necessarily consist only from the state measurement. Depending on the MPC problem formulation, the parameter theta is extended by several other variables. When reference tracking is achieved, then the parameter theta is extended by the reference values as well. Same goes for the measured/estimated disturbances. Furthermore, when the Δu variable is introduced to the MPC formulation, θ

parameter must be extended by variable denoted as $u(t - T_\mathrm{s})$, as suggested by e.g. (3.10).

3.4 Concluding Remarks

In this chapter was presented well known theory of the model predictive control strategy. Formulations based on difference equations and state space equations were presented. Individual MPC formulations were modified to enforce the tracking objective. Moreover, the approach fo disturbance modeling or offset-free control has been discussed to further strengthened the ability of the MPC to remove any offset in reference tracking.

If the collection of presented optimal control strategies would be implemented to control processes directly, one would gain significant improvements over traditional strategies. In the next chapter we will present the alternative, called reference governor control schemes. In these schemes we allow to keep current control infrastructure, on top of which an optimization layer improves the total performance.

CHAPTER 4

MPC-based Reference Governors

The reference governor is a control strategy which objective is to modify the user given reference r in order to improve the quality and safety of a system, which already consist of a primary controller. We will refer to this control strategy as an MPC-based reference governor (MPC-RG). This secondary layer of control serves as a supervisory layer to the primary control layer, cf. Fig. 4.1. The primary control layer consists of low level controllers, which provides the actual control inputs and stabilizes the process and provide some tracking performance.

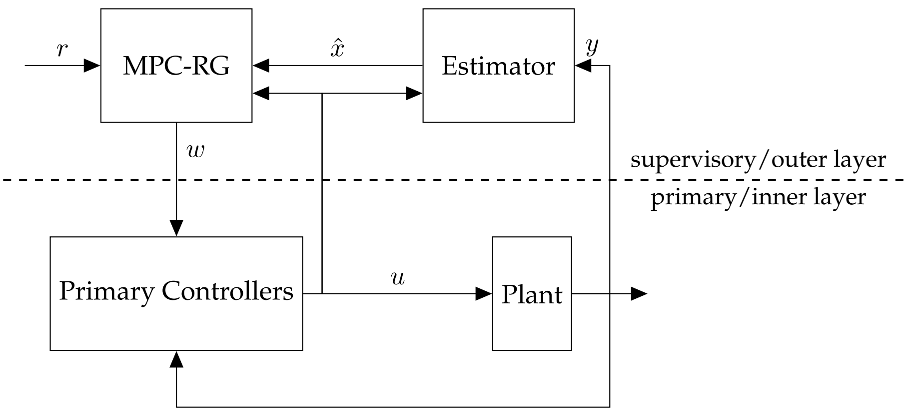


Figure 4.1: Conceptual diagram of the optimization-based reference governor scheme.

Objectives of the reference governor are to improve tracking performance while guaranteeing rigorous constraint satisfaction. The objectives are achieved by optimizing setpoints for the primary layer of controllers. Note, that user will not longer supply setpoints to the primary controllers, but it will be provided to the reference governor, which will then calculate setpoints for the primary layer. Such an arrangement has two key advantages. First, since the setpoints fro the primary layer are provided in optimal fashion, second if the MPC-RG fails to provide the setpoint, the inner layer still maintains the stability of the operation.

The reference governor is formulated as an MPC-based control strategy. The MPC-based reference governor can be formally given as

$$
\begin{aligned}
\min_{W} \quad & \ell_{\mathrm{N}}(x_N) + \sum_{k=0}^{N-1} \ell\left(x_k, y_k, u_k, w_k\right) && && \text{(4.1a)}\\
\text{s.t.} \quad & x_{k+1} = f(x_k, u_k), && k \in \mathbb{N}_0^{N-1}, && \text{(4.1b)}\\
& y_k = g(x_k, u_k), && k \in \mathbb{N}_0^{N-1}, && \text{(4.1c)}\\
& u_k = h(x_k, w_k), && k \in \mathbb{N}_0^{N-1}, && \text{(4.1d)}\\
& u_k \in \mathcal{U}, && k \in \mathbb{N}_0^{N-1}, && \text{(4.1e)}\\
& x_k \in \mathcal{X}, && k \in \mathbb{N}_0^{N-1}, && \text{(4.1f)}\\
& y_k \in \mathcal{Y}, && k \in \mathbb{N}_0^{N-1}, && \text{(4.1g)}\\
& w_k \in \mathcal{W}, && k \in \mathbb{N}_0^{N-1}, && \text{(4.1h)}\\
& x_0 = x(t) && && \text{(4.1i)}
\end{aligned}
$$

where $u \in \mathbb{R}^{n_\mathrm{u}}$, $x \in \mathbb{R}^{n_\mathrm{x}}$, $y \in \mathbb{R}^{n_\mathrm{y}}$, are control variables, system states, process variables, respectively. By $r \in \mathbb{R}^{n_\mathrm{y}}$ is denoted the user defined reference and the $w \in \mathbb{R}^{n_\mathrm{y}}$, represents the set point for the primary layer of controller. The optimization is solved to obtain the sequence

$$
W = \begin{bmatrix} w_0^\intercal & \dots & w_{N-1}^\intercal \end{bmatrix}^\intercal \tag{4.2}
$$

while enforcing constraints (4.1b)-(4.1h). The main difference between the MPC-based reference governor in (4.1) and the MPC in (3.1) is the prediction model $h(x_k, u_k, w_k)$ in (4.1d) which represent the model of the primary controller. Furthermore, we allow to constrain also the setpoint w in (4.1h).

REMARK 4.1. Without loss of generality we assume, that the state of the process can be directly measured, as per the initial condition (4.1i). However, the states are often abstract, so they can not be measured directly, hence we include the estimator in the control scheme, cf. Fig 4.1. This estimator can be design as a time-varying Kalman Filter as discussed in the Section 3.2. □

Challenges in formulating the MPC-based reference governor lies in the type of the $h(\cdot)$ in (4.1d). In subsequent sections, we will show how the MPC-based governor can be constructed if the primary layer consist of

1. **PID controllers**, discussed in the Section 4.1 and published in [Klaučo et al., 2017, Klaučo and Kvasnica, 2017],

2. **relay-based controllers**, discussed in the Section 4.2, and published in [Drgoňa et al., 2015],

3. **local MPC controllers**, elaborated in the Section 4.3, and published in Holaza et al. [2017].

In all three cases we will first derive the model of the primary layer with the process, i.e., the closed-loop system, hence we will identify equations $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$. This model will then serve as a prediction model for the MPC-based reference governor.

4.1 Systems with Set of PID Controllers

A situation where the process is controlled primary by a set of PID controllers is very common in the industry. This section is devoted to the mathematical modeling of such closed-loop systems which consists of a process and an inner controller in a form of a set of PID controllers. We assume that the process is described by the linear model (3.12). Overall scheme of such a control strategy can be seen on the Fig. 4.2.

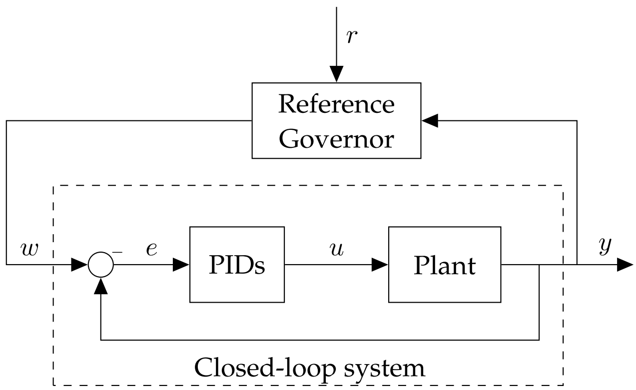


Figure 4.2: Closed-loop system with MPC-based reference governor control strategy.

Depending on the particular choice of the process modeling, the structure of the MPC-RG will change. In this chapter we will formulate the MPC-based governor control problems for three main cases. First, in the Section 4.1.1, we consider a SISO process characterized by a transfer function. Next, the Section 4.1.2 is devoted to two MIMO cases.

4.1.1 SISO Case

If the controlled system has one input and a single output, we can describe this system by a transfer function of the form

$$G(s) = \frac{Y(s)}{U(s)}, \tag{4.3}$$

where $U(s)$ represents the Laplace transform of manipulated variable $u(t)$ in the s-domain, and the variable $Y(s)$ is the s-domain counterpart of the output signal $y(t)$. For such a system exist a PID controller in a form of a single transfer function

$$R(s) = \frac{U(s)}{E(s)}, \tag{4.4}$$

in which $E(s)$ is the Laplace transform of the control error $e(t)$ defined as $e(t) = w(t) - y(t)$. By $w(t)$ we denote the set point value for the PID controller. Since we are dealing with SISO case, then $w(t), e(t), u(t), y(t) \in \mathbb{R}$ If we apply the rules of transfer function algebra, we obtain the model of the closed-loop system given by

$$G_{\text{cl}}(s) = \frac{Y(s)}{W(s)} = \frac{G(s)R(s)}{1 + G(s)R(s)}. \tag{4.5}$$

In the MPC design a discrete time version of the model is considered, hence the transfer function $G_{\text{cl}}(s)$ from (4.5) is discretized. We can use $z-$transform to obtain the discrete time version of $G_{\text{cl}}(s)$, which we will denote as $H_{\text{cl}}(z^{-1})$ for a given sampling instant T_{s}. Furthermore, the numerator and the denominator of $H_{\text{cl}}(z^{-1})$ can be written as polynomials of z^{-1}, leading to

$$H_{\text{cl}}(z^{-1}) = \frac{Y(z^{-1})}{W(z^{-1})} = \frac{\sum\limits_{j=0}^{m} b_j z^{-j}}{\sum\limits_{i=0}^{n+1} a_i z^{-i}}. \tag{4.6}$$

If the discrete time transfer function is proper, i.e., $m \leq n$, we can rewrite the transfer function into a recursive definition as follows

$$y(t+1) = \frac{1}{a_0} \left(-\sum_{i=1}^{n} a_i y(t-i+1) + \sum_{j=0}^{m} b_j w(t-j+1) \right). \tag{4.7}$$

The model in (4.7) represents exactly the prediction equation as in the Section 3.1.1. The MPC-based reference governor has following form

$$\min_{u_1,\ldots,u_N} \quad \sum_{k=1}^{N} \ell_{\text{io}}\left(y_k, u_k, w_k\right) \tag{4.8a}$$

$$y_{k+1} = f_{\text{y}}(y_k, \ldots, y_{k-n}, w_k, \ldots, w_{k-m}), \quad k \in \mathbb{N}_0^{N-1} \tag{4.8b}$$

$$y_{\min} \leq y_k \leq y_{\max}, \quad k \in \mathbb{N}_0^{N-1} \tag{4.8c}$$

$$\theta_0 = \theta(t), \tag{4.8d}$$

with the objective function

$$\begin{aligned}\ell_{\text{io}}(\cdot) = {} & \|Q_{\text{ry}}\left(r_k - y_k\right)\|_2 + \|Q_{\text{rw}}\left(r_k - w_k\right)\|_2 + \\ & \|Q_{\text{wy}}\left(w_k - y_k\right)\|_2 + \|Q_{\text{dw}}\left(w_k - w_{k-1}\right)\|_2. \end{aligned} \tag{4.9}$$

The objective function is extended by several term compared to the original ℓ_{io} in (3.11). First term, penalizes the difference between the measured output and the user-defined reference r. Second term penalizes the enforce convergence of the shaped reference. Third penalization term provides that the predicted evolution of the closed-loop system converges to the shaped reference. Last term dampens the fluctuations in the evolution of the shaped reference throughout prediction window. The penalty matrices $Q_{\text{ry}} \succeq 0$, $Q_{\text{rw}} \succeq 0$, $Q_{\text{wy}} \succeq 0$, and $Q_{\text{dw}} \succeq 0$ allow the designer to give different priorities to individual properties of the reference governor.

The vector of initial conditions (4.8d) is for this case is expressed as

$$\theta = \begin{bmatrix} y(t) & \ldots & y(t-nT_{\text{s}}) & w(t-T_{\text{s}}) & \ldots & w(t-mT_{\text{s}}) & r_1 & \ldots & r_N \end{bmatrix}^{\mathsf{T}}. \tag{4.10}$$

REMARK 4.2. The length of the parameter θ can be decreased by $N-1$ elements once we do not provide the future user-defined reference profile. □

The transfer function based prediction equations shown as in (4.7) is suitable to use if we are dealing with SISO systems. On the other hand, some applications require the use of state space approach. The state space based modeling can be easily extended also to a MIMO case, which is covered in the next section.

4.1.2 MIMO Case

In the modeling of the MIMO closed-loop system we must distinguish between two principal cases. For both cases a closed-loop model is derived using state space modeling. Findings presented especially in the Section 4.1.2.1 are used to formulate MPC strategy mentioned in Chapter 6.

4.1.2.1 Symmetric Case

This case assumes the fact that the number of controlled outputs is equal to the number of inputs. This also means that the number of set points is coincidental to the number of outputs. Formally, such a condition is expressed as

$$n_{\mathrm{y}} = n_{\mathrm{u}} = n_{\mathrm{w}} = p,$$

where p is the number of PID controllers. Control scheme is visualized in the Fig. 4.3.

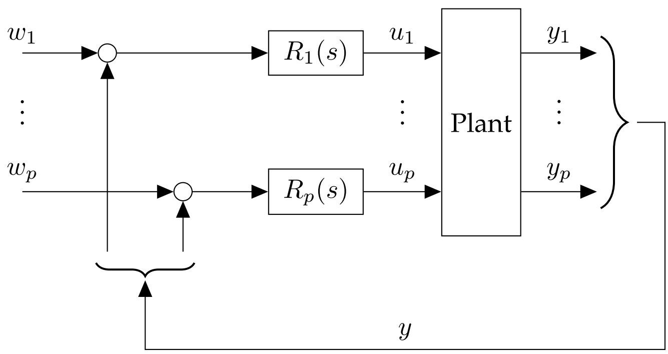


Figure 4.3: Control scheme of a MIMO PID control setup.

Consider that the plant model is represented by the state space equations from (3.12). Similarly to the plant model, we can obtain the state space model of PID controllers $R_{1,\dots,p}(s)$. The compact representation of such a model can be expressed as

$$x_{\mathrm{r}}(t + T_{\mathrm{s}}) = A_{\mathrm{r}} x_{\mathrm{r}}(t) + B_{\mathrm{r}} e(t), \tag{4.11a}$$

$$u(t) = C_{\mathrm{r}} x_{\mathrm{r}}(t) + D_{\mathrm{r}} e(t), \tag{4.11b}$$

where the variable x_{r} denotes the aggregated vector of states of individual PID controllers, i.e.,

$$x_{\mathrm{r}} = \begin{bmatrix} x_{\mathrm{r},1}^{\intercal} & x_{\mathrm{r},2}^{\intercal} & \dots & x_{\mathrm{r},p}^{\intercal} \end{bmatrix}^{\intercal}, \tag{4.12}$$

where variable $x_{\mathrm{r},1}$ represents the states of the first PID controller, etc. Since a PID controller is represented in general by a second order transfer function, hence $x_{\mathrm{r}} \in \mathbb{R}^{2p}$.

Remark 4.3. The construction of the compact state space model of the PID controllers in (4.11) can be easily extended to accommodate also any polynomial controllers. The only difference will be increased size of the vector x_{r} in (4.12). □

In order to derive the state space model of the closed-loop system we must first formulate an open-loop model. Such a model is easily formed by combing (3.12) with (4.11), yielding a combined state vector

$$\widetilde{x} = \begin{bmatrix} x_\mathrm{r} \\ x \end{bmatrix}. \tag{4.13}$$

Furthermore, the expression for $u(t)$ from (4.11b) is inserted into state equation (3.12a) and to output equation (3.12b). Then the state space model of the open-loop system is given as

$$\widetilde{x}(t+T_\mathrm{s}) = A_\mathrm{OL}\widetilde{x}(t) + B_\mathrm{OL}e(t), \tag{4.14a}$$
$$u(t) = C_\mathrm{OL,u}\widetilde{x}(t) + D_\mathrm{OL,u}e(t), \tag{4.14b}$$
$$y(t) = C_\mathrm{OL,y}\widetilde{x}(t) + D_\mathrm{OL,y}e(t), \tag{4.14c}$$

where,

$$A_\mathrm{OL} = \begin{bmatrix} A_\mathrm{r} & 0 \\ BC_\mathrm{r} & A \end{bmatrix}, \tag{4.15a}$$
$$B_\mathrm{OL} = \begin{bmatrix} B_\mathrm{r} \\ BD_\mathrm{r} \end{bmatrix}, \tag{4.15b}$$
$$C_\mathrm{OL,u} = \begin{bmatrix} C_\mathrm{r} & 0 \end{bmatrix}, \tag{4.15c}$$
$$D_\mathrm{OL,u} = D_\mathrm{r}, \tag{4.15d}$$
$$C_\mathrm{OL,y} = \begin{bmatrix} DC_\mathrm{r} & C \end{bmatrix}, \tag{4.15e}$$
$$D_\mathrm{OL,y} = DD_\mathrm{r}. \tag{4.15f}$$

Next, combining the expression for tracking error $e(t)$ which is given as

$$e(t) = w(t) - y(t), \tag{4.16}$$

with the equation (4.14c) yields

$$y(t) = \left(I + D_\mathrm{OL,y}\right)^{-1} C_\mathrm{OL,y}\widetilde{x}(t) + \left(I + D_\mathrm{OL,y}\right)^{-1} D_\mathrm{OL,y}w(t). \tag{4.17}$$

After inserting (4.16) and (4.17) into (4.14) we obtain following matrix expressions

$$A_{\mathrm{CL}} = A_{\mathrm{OL}} - B_{\mathrm{OL}} \left(I + D_{\mathrm{OL,y}}\right)^{-1} C_{\mathrm{OL,y}}, \tag{4.18a}$$
$$B_{\mathrm{CL}} = B_{\mathrm{OL}} - B_{\mathrm{OL}} \left(I + D_{\mathrm{OL,y}}\right)^{-1} D_{\mathrm{OL,y}}, \tag{4.18b}$$
$$C_{\mathrm{CL,u}} = C_{\mathrm{OL,u}} - D_{\mathrm{OL,u}} \left(I + D_{\mathrm{OL,y}}\right)^{-1} C_{\mathrm{OL,u}}, \tag{4.18c}$$
$$D_{\mathrm{CL,u}} = D_{\mathrm{OL,u}} - D_{\mathrm{OL,u}} \left(I + D_{\mathrm{OL,y}}\right)^{-1} D_{\mathrm{OL,u}}, \tag{4.18d}$$
$$C_{\mathrm{CL,y}} = \left(I + D_{\mathrm{OL,y}}\right)^{-1} C_{\mathrm{OL,y}}, \tag{4.18e}$$
$$D_{\mathrm{CL,y}} = \left(I + D_{\mathrm{OL,y}}\right)^{-1} D_{\mathrm{OL,y}}, \tag{4.18f}$$

which are then arranged in a familiar fashion into a full-fledged state space model of the closed-loop system of the form

$$\widetilde{x}(t + T_{\mathrm{s}}) = A_{\mathrm{CL}}\widetilde{x}(t) + B_{\mathrm{CL}}w(t), \tag{4.19a}$$
$$u(t) = C_{\mathrm{CL,u}}\widetilde{x}(t) + D_{\mathrm{CL,u}}w(t), \tag{4.19b}$$
$$y(t) = C_{\mathrm{CL,y}}\widetilde{x}(t) + D_{\mathrm{CL,y}}w(t). \tag{4.19c}$$

Model in (4.19) allows us to simulate responses of the states $\widetilde{x}(t)$ based on (4.19a), the evolution of the manipulated variable $u(t)$ in (4.19b) as well as the output profile $y(t)$ in (4.19c) given the reference $w(t)$. Compared to the transfer function based approach in Section 4.1.1, we can easily access also control inputs.

The MPC-based reference governor is casted as a constrained finite time optimization problem

$$\min_{u_0,\ldots,u_{N-1}} \quad \ell_{\mathrm{N}}(x_N) + \sum_{k=0}^{N-1} \ell_{\mathrm{ss}}\left(y_k, u_k, w_k\right) \tag{4.20a}$$
$$\text{s.t.} \quad \widetilde{x}_{k+1} = A_{\mathrm{CL}}x_k + B_{\mathrm{CL}}w_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20b}$$
$$u_k = C_{\mathrm{CL},u}\widetilde{x}_k + D_{\mathrm{CL},u}w_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20c}$$
$$y_k = C_{\mathrm{CL},y}\widetilde{x}_k + D_{\mathrm{CL},y}w_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20d}$$
$$u_{\min} \leq u_k \leq u_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20e}$$
$$x_{\min} \leq x_k \leq x_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20f}$$
$$y_{\min} \leq y_k \leq y_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.20g}$$
$$x_0 = x(t), u_{-1} = u(t - T_{\mathrm{s}}), w_{-1} = w(t - T_{\mathrm{s}}), \tag{4.20h}$$

with objective function defined as

$$\begin{aligned}\ell_{\mathrm{ss}}(\cdot) = {} & \|Q_{\mathrm{ry}}\left(r_k - y_k\right)\|_2^2 + \|Q_{\mathrm{rw}}\left(r_k - w_k\right)\|_2^2 + \|Q_{\mathrm{wy}}\left(w_k - y_k\right)\|_2^2 + \\ & \|Q_{\mathrm{dw}}\left(w_k - w_{k-1}\right)\|_2^2 + \|Q_{\mathrm{du}}\left(u_k - u_{k-1}\right)\|_2^2.\end{aligned} \tag{4.21}$$

The structure of the objective function in (4.21) is similar to $\ell_{\text{io}}(\cdot)$ in (4.9), except of the penalization of the control slew rates. Note, that since the manipulated variable u is no longer the optimization variable, we do not necessarily require a penalization of this variable in the objective function.

In the next subsection we will focus on a more general case, when the number of manipulated variables does not coincide with the number of controlled variables.

4.1.2.2 General MIMO Case

The characteristic of this second principal case is that the number of manipulated variables differs from the number of controlled signals. Here, we must distinguish between two scenarios. First, if $n_{\text{u}} < n_{\text{y}}$ then the system is not fully controllable. Second case is, if $n_{\text{u}} > n_{\text{y}}$, then during control may arise a problem with maintaining the offset-free qualities of the control performance. However, in the each of these two sub-cases a pre-compensator must be included in the control scheme in order to compensate between mismatching number of control variables and manipulated variables. Such a control scheme is illustrated in the Fig. 4.4. In such a scheme the individual PID controllers do not directly provide the control action $u(t)$, however, they provide an auxiliary control action, denoted as $v(t)$, which is supplied to the pre-compensator. The actual control action $u(t)$ is provided by the pre-compensator filter. The design of the pre-compensator filter is usually realized in s-domain [Skogestad and Postlethwaite, 2005, ch. 3], i.e., as a set of transfer functions, generally denoted by $\mathbf{T}(s)$. The pre-compensator is a matrix of transfer functions given as

$$\mathbf{T}(s) = \begin{bmatrix} T_{1,1}(s) & T_{1,2}(s) & \cdots & T_{1,n_{\text{y}}}(s) \\ T_{2,1}(s) & T_{2,2}(s) & \cdots & T_{2,n_{\text{y}}}(s) \\ \vdots & \ddots & \ddots & \vdots \\ T_{n_{\text{u}},1}(s) & T_{n_{\text{u}},2}(s) & \cdots & T_{n_{\text{u}},n_{\text{y}}}(s) \end{bmatrix}. \tag{4.22}$$

The matrix $\mathbf{T}(s)$ can be converted into one state space model, denoted as

$$x_{\text{T}}(t + T_{\text{s}}) = A_{\text{T}} x_{\text{T}}(t) + B_{\text{T}} v(t), \tag{4.23a}$$

$$u(t) = C_{\text{r}} x_{\text{T}}(t) + D_{\text{T}} v(t), \tag{4.23b}$$

where the variable x_{T} denotes the utilized vector of states, which originates in converting each of the single transfer function in (4.22). Recall, that signal $v(t)$ is the actual output from individual PID controllers and the output from this state

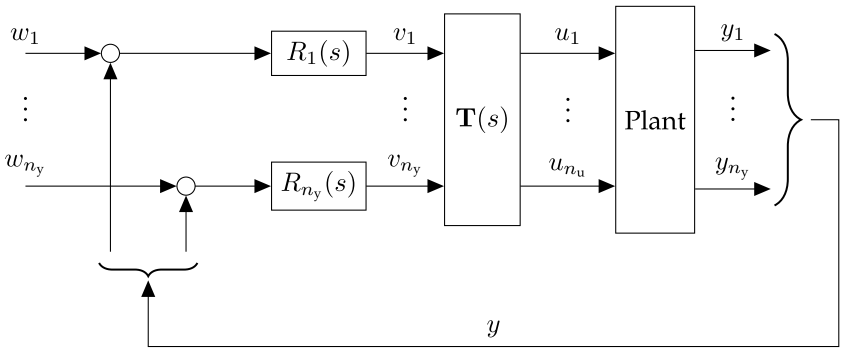


Figure 4.4: Control scheme of a MIMO PID control setup, where $\mathbf{G}(s)$ represents the transfer function matrix model of the MIMO system and the pre-compensator is denoted by the matrix $\mathbf{T}(s)$.

space model is the manipulated variable $u(t)$. In order to derive the model of the closed-loop system with the filter $T(s)$, we will first derive the combined model of the filter (4.22) and the plant model (3.12). We will substitute the variable $u(t)$ in the model (3.12) for the expression in (4.23b). Such a substitution results in an aggregated model

$$x_T(t+T_s) = A_T x_T(t) + B_T v(t), \tag{4.24a}$$
$$x(t+T_s) = Ax(t) + B\left(C_T x_T(t) + D_T v(t)\right), \tag{4.24b}$$
$$y(t) = Cx(t) + D\left(C_T x_T(t) + D_T v(t)\right), \tag{4.24c}$$

which can be restructured to a matrix form

$$\begin{bmatrix} x_T(t+T_s) \\ x(t+T_s) \end{bmatrix} = \begin{bmatrix} A_T & 0 \\ BC_T & A \end{bmatrix} \begin{bmatrix} x_T(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} B_T \\ BD_T \end{bmatrix} v(t), \tag{4.25a}$$
$$y(t) = \begin{bmatrix} DC_T & C \end{bmatrix} \begin{bmatrix} x_T(t) \\ x(t) \end{bmatrix} + DD_T v(t). \tag{4.25b}$$

Next, we can treat the model in (4.25) as the main system, which is controlled by a set of PID controllers and we can apply matrix manipulations as in (4.14)-(4.19). Now, we have obtained a state space model of the closed-loop system which also includes a pre-compensator filter. The MPC-based strategy can be then designed exactly in the same fashion as is expressed in (4.20) with the objective function (4.21).

REMARK 4.4. The number of states in such a system can increase dramatically due to the structure of the filter $\mathbf{T}(s)$. Such an increase on system states can negatively effect the complexity of the MPC control strategy. □

The MPC-based reference governor based on a transfer function modeling (Section 4.1.1) of the closed-loop system was experimentally applied, and the results of these experiments are addressed in detail in the Chapter 5. Moreover, we have also applied the MPC-RG based on the state space model of the closed-loop system containing three individual PI controllers, and these results are covered in the Chapter 6.

4.2 Systems with Relay-based Controllers

Relay-based control is also known under the term on-off control or a bang–bang control. It is one of the oldest types of controllers. The operation of such a controller is limited to providing either minimum, u_{min}, or maximum, u_{max}, value of the control action. These min/max values correspond to on/off states of the relay, since usually the relay controller does not provide the actual control action. The change between providing the maximum value is triggered by the measured variable crossing a threshold γ. Depending on particular application, the maximum value can be applied of the threshold is crossed in the negative direction and vise-versa. The operation of the relay-based controller is depicted on the Fig. 4.5. Theoretical formulations presented in this section are adopted

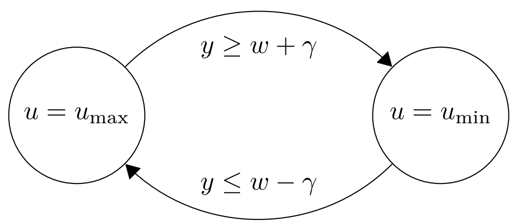


Figure 4.5: Relay-based controller.

from [Drgoňa, Klaučo, and Kvasnica, 2015], where specific MPC-RG for on/off controller was designed to improve the behavior of a thermostatically controlled temperature in building. Here, in this section we generalized this approach to an arbitrary on/off controller.

4.2.1 Model of the Relay-based Controllers

The model of the relay based controller, represented by the finite state machine by Fig. 4.5, can be captured by introducing a binary state variable $z \in \{0, 1\}$. Then $z = 1$ represents the "on" state, or the state when u_{max} is applied, while the "off" state corresponds to $z = 0$. The dynamical evolution of the state z is given by a state equation

$$z(t + T_\mathrm{s}) = f_\mathrm{z}\big(z(t), y(t), w(t)\big), \tag{4.26}$$

where $z(t)$ is the current value of the binary state, $y(t)$ is current measurement a the $w(t)$ is the setpoint provided to the relay-based controller. The state update function $f_\mathrm{z}(\cdot)$ is given by

$$f_\mathrm{z}\big(z(t), y(t), w(t)\big) = \begin{cases} 1 & \text{if} \quad (z = 1 \ \wedge \ \neg\,(y \geq r + \gamma)) \vee \\ & \qquad (z = 0 \ \wedge \ (y \leq r - \gamma)) \\ 0 & \text{otherwise} \end{cases} \tag{4.27}$$

where the "$\neg$" denotes the logic negation, "$\wedge$" is the logic conjunction and "$\vee$" stands for the logic disjunction. Once the binary state of the thermostat is available, the value of the manipulated variable provided to the process is given by

$$u = \begin{cases} u_\text{max} & \text{if} \quad z = 1 \\ u_\text{min} & \text{if} \quad z = 0. \end{cases} \tag{4.28}$$

Note, that the relay-based controller does not have to necessarily switch between u_max and u_min. These values may be chosen arbitrarily based on the needs of the particular technology, they even can be time-dependent.

4.2.2 Reference Governor Synthesis

The synthesis of the MPC-based reference governor presented in this section is adopted and generalized based on paper by Drgoňa et al. [2015]. The closed-loop model with a relay-based controller is given by a linear discrete time state space model as in (3.12) and by a control law given by the binary state update equation (4.26). The control scheme with the inner loop is shown in the Fig. 4.6.

In the MPC-based design, the optimal profile of modulated references w is obtained over a prediction time frame N based on the state measurements. Specifically, the optimal sequence $w_{0,\ldots,N-1}$ can be obtained by solving follow-

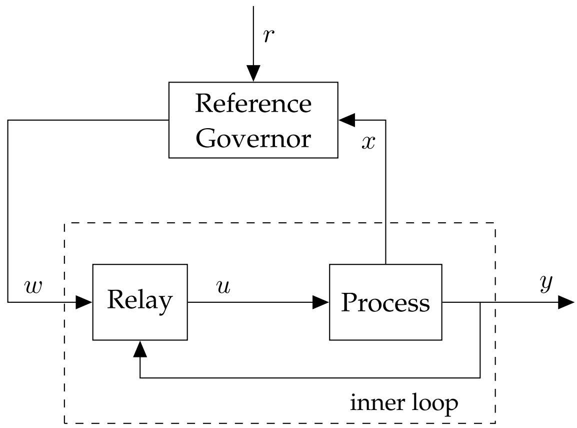


Figure 4.6: Reference governor scheme with a relay-based controller.

ing optimal control problem

$$\min_{W} \ \ell_z \left(x_k, y_k, u_k, w_k\right) \tag{4.29a}$$

$$\text{s.t.} \ x_{k+1} = Ax_k + Bu_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29b}$$

$$y_k = Cx_k + Du_k, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29c}$$

$$x_{\min} \leq x_k \leq x_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29d}$$

$$y_{\min} \leq y_k \leq y_{\max}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29e}$$

$$u_k = \begin{cases} u_{\max} & \text{if } z_k = 1 \\ u_{\min} & \text{otherwise.} \end{cases}, \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29f}$$

$$z_{k+1} = f_z\big(z_k, y_k, w_k\big), \qquad k \in \mathbb{N}_0^{N-1} \tag{4.29g}$$

$$x_0 = x(t), z_0 = z(t). \tag{4.29h}$$

Again, the x_k denotes the k-th step prediction of the state vector obtained though (4.29b) based on (4.29h). The dynamical behavior of the on-off evolves according to (4.29g) and (4.29f). By employing a relay-based controller we inherently can not achieve a zero offset while tracking the reference. The choice of the objective function is this case is depends on particular application. One of the choice is to include a term $\|Q_{\text{ry}} r_k - y_k\|_2^2$ which enforces the reference tracking part, however in this case it will result in oscillating behavior. Individual applications may require a direct penalization of the manipulated variable, $\|Q_{\text{u}} u_k\|_2^2$ since usually this signal is directly related to the energy spent to achieve the control objectives.

Unfortunately, the problem (4.29) cannot be immediately solved since it contains

IF-THEN rules in (4.29f) and (4.29g) via (4.27). Therefore, we show that these constraints can be rewritten into a set of inequalities that are linear in the decision variables. However, since the state z is a binary variable, the entire reformulated control problem will become a non-convex mixed-integer quadratic problem (MIQP).

4.2.3 Mixed-Integer Problem Formulation

First, we start by rewriting the IF-THEN rule in (4.28) to a set of logic rules

$$[z = 1] \Leftrightarrow [u = u_{\text{max}}], \tag{4.30a}$$
$$[z = 0] \Leftrightarrow [u = u_{\text{min}}], \tag{4.30b}$$

where the "$\Leftrightarrow$" denotes the logic equivalence. Next, we rewrite (4.30) into a set of inequalities using propositional logic Williams [1993]. Then (4.30) hold if and only if z and u satisfy

$$-M_i(1-z) \leq u_i - u_{\text{max},i} \leq M_i(1-z), \qquad \forall i \in \mathbb{N}_1^{n_\text{u}}, \tag{4.31a}$$
$$-M_i z \leq u_i - u_{\text{min},i} \leq M_i z, \qquad \forall i \in \mathbb{N}_1^{n_\text{u}}, \tag{4.31b}$$

where $M_{1,\dots,n_\text{u}}$ are sufficiently large scalars. The value M_i is often chosen automatically by individual software tools (like YALMIP in MATLAB), however if we need to choose it manually, we can adopt following expression

$$M_i = \max\{|u_{\text{min},i}|, |u_{\text{max},i}|\}, \quad \forall i \in \mathbb{N}_1^{n_\text{u}}. \tag{4.32}$$

In order further introduce the reader to the logic reformulation, let us show the results in $z = 0$, then

$$-M_i \leq u_i - u_{\text{max},i} \leq M_i, \qquad \forall i \in \mathbb{N}_1^{n_\text{u}}, \tag{4.33a}$$
$$0 \leq u_i - u_{\text{min},i} \leq 0, \qquad \forall i \in \mathbb{N}_1^{n_\text{u}}, \tag{4.33b}$$

which can be further expanded to

$$-M_i \leq u_i - u_{\text{max},i} \leq M_i \qquad \to \begin{cases} u_i \geq u_{\text{max},i} - M_i \\ u_i \leq u_{\text{max},i} + M_i \end{cases}, \tag{4.34a}$$
$$0 \leq u_i - u_{\text{min},i} \leq 0 \qquad \to \begin{cases} u_i \geq u_{\text{min},i} \\ u_i \leq u_{\text{min},i} \end{cases}, \tag{4.34b}$$

enforced for $i = \{1, \dots, n_\text{u}\}$. The inequalities in (4.34a) reads, that the value u_i is bounded from the bottom by a large negative number and the a large positive number from the top. But, the second set of inequalities (4.34b) points to $u_i = u_{\min,i}$, since from the bottom and from the top the variable u_i is bounded by the same expression $u_{\min,i}$. Same logic can be applied for the case when $z = 1$, but we will obtain inverted scenario compared to the (4.34), hence we will enforce logical rules in (4.30) via set of linear inequalities.

The translation of the binary state equation $f_{\mathrm{z}}(\cdot,\cdot,\cdot)$ as in (4.27) starts by rewriting it again to a set of logic equivalence rules

$$f_{\mathrm{z}}(z, T, r) = \begin{cases} 1 & \text{if } (z \ \wedge \ \neg\delta_{\mathrm{a}}) \vee (\neg z \ \wedge \ \delta_{\mathrm{b}}) \\ 0 & \text{otherwise} \end{cases} \tag{4.35}$$

where δ_{a} and δ_{b} are binary variables that indicate whether the measurement y is above the upper threshold (i.e., $y \geq w + \gamma$), or below the bottom threshold (i.e., $y \leq w - \gamma$). Specifically,

$$[\delta_{\mathrm{a}} = 1] \Leftrightarrow [y \geq w + \gamma], \tag{4.36a}$$
$$[\delta_{\mathrm{b}} = 1] \Leftrightarrow [y \leq w - \gamma]. \tag{4.36b}$$

Next we rewrite (4.36) into a set of inequalities using propositional logic as have done with the first IF-THEN rule.

LEMMA 4.1 (WILLIAMS [1993]). Consider the statement

$$[\delta = 1] \Leftrightarrow [g(v) \leq 0], \tag{4.37}$$

where $\delta \in \{0, 1\}$ is a binary variable, v is a vector of continuous variables, and $g(\cdot)$ is any function. Then (4.37) holds if and only if δ and v satisfy

$$g(v) \leq M(1 - \delta), \tag{4.38a}$$
$$g(v) \geq \epsilon + (m - \epsilon)\delta, \tag{4.38b}$$

where M is a sufficiently large scalar, m is a sufficiently small scalar, and $\epsilon > 0$ is the machine precision. For the scalars M and m holds

$$M = \max\left(g(v)\right) \tag{4.39a}$$
$$m = \min\left(g(v)\right) \tag{4.39b}$$
$$\tag{4.39c}$$

We can apply Lemma 4.1 to (4.36a) as follows. Take $v = [y^{\intercal}, w^{\intercal}]^{\intercal}$ and let $g(v) := w + \gamma - y$. Then (4.36a) is equivalent to

$$w + \gamma - y \leq M_{\mathrm{a}}(1 - \delta_{\mathrm{a}}), \tag{4.40a}$$
$$w + \gamma - y \geq \epsilon + (m_{\mathrm{a}} - \epsilon)\delta_{\mathrm{a}}. \tag{4.40b}$$

With $g(v) := y - w + \gamma$, (4.36b) is satisfied if and only if

$$y - w + \gamma \leq M_{\text{b}}(1 - \delta_{\text{b}}), \tag{4.41a}$$
$$y - w + \gamma \geq \epsilon + (m_{\text{b}} - \epsilon)\delta_{\text{b}}. \tag{4.41b}$$

Next, we rewrite the IF condition of (4.35), i.e., $(z \ \wedge \ \neg\delta_{\text{a}}) \vee (\neg z \ \wedge \ \delta_{\text{b}})$, into inequalities in binary variables z, δ_{a}, and δ_{b}. Introduce the substitutions

$$\delta_1 = \ z \ \wedge \ \neg\delta_{\text{a}}, \tag{4.42a}$$
$$\delta_2 = \ \neg z \ \wedge \ \delta_{\text{b}}, \tag{4.42b}$$
$$\delta_3 = \ \delta_1 \vee \delta_2, \tag{4.42c}$$

where δ_1, δ_2, δ_3 are new binary variables. It is easy to verify that (4.42a) is equivalent to

$$\delta_1 \leq z, \tag{4.43a}$$
$$\delta_1 \leq (1 - \delta_{\text{a}}), \tag{4.43b}$$
$$z + (1 - \delta_{\text{a}}) \leq 1 + \delta_1, \tag{4.43c}$$

which uses the fact that the negation of a binary variable δ_{a} can be equivalently written as $1 - \delta_{\text{a}}$. Using the same approach, (4.42b) can be rewritten as

$$\delta_2 \leq (1 - z), \tag{4.44a}$$
$$\delta_2 \leq \delta_{\text{b}}, \tag{4.44b}$$
$$(1 - z) + \delta_{\text{b}} \leq 1 + \delta_2, \tag{4.44c}$$

which is a set of inequalities that are linear in δ_2, z, and δ_{b}. Finally, (4.42c) is equivalent to

$$\delta_3 \geq \delta_1, \tag{4.45a}$$
$$\delta_3 \geq \delta_2, \tag{4.45b}$$
$$\delta_3 \leq \delta_1 + \delta_2. \tag{4.45c}$$

Therefore δ_3 equivalently models the satisfaction of the IF condition in (4.35). Then $f_{\text{z}}(z, T, r)$ in (4.27) can be rewritten as

$$f_{\text{z}}(z, T, r) = \delta_3. \tag{4.46}$$

With (4.28) rewritten per (4.30) and $f_{\text{z}}(\cdot, \cdot, \cdot)$ in (4.27) being equivalent to (4.40)–

(4.46), problem (4.29) can be equivalently stated as

$$\min_{W} \sum_{k=0}^{N-1} \ell_{\mathrm{z}}(u_k, y_k, w_k, r_k) \tag{4.47a}$$
$$\text{s.t. } x_{k+1} = Ax_k + Bu_k, \tag{4.47b}$$
$$y_k = Cx_k + Du_k, \tag{4.47c}$$
$$u_{k,i} \geq u_{\max,i} - M_i(1 - z_k) \tag{4.47d}$$
$$u_{k,i} \leq u_{\max,i} + M_i(1 - z_k) \tag{4.47e}$$
$$u_{k,i} \geq u_{\min,i} - M_i z_k \tag{4.47f}$$
$$u_{k,i} \leq u_{\min,i} + M_i z_k \tag{4.47g}$$
$$w_k + \gamma - y_k \leq M_{\mathrm{a}}(1 - \delta_{\mathrm{a},k}), \tag{4.47h}$$
$$w_k + \gamma - y_k \geq \epsilon + (m_{\mathrm{a}} - \epsilon)\delta_{\mathrm{a},k}, \tag{4.47i}$$
$$y_k - w_k + \gamma \leq M_{\mathrm{b}}(1 - \delta_{\mathrm{b},k}), \tag{4.47j}$$
$$y_k - w_k + \gamma \geq \epsilon + (m_{\mathrm{b}} - \epsilon)\delta_{\mathrm{b},k}, \tag{4.47k}$$
$$\delta_{1,k} \leq z_k, \tag{4.47l}$$
$$\delta_{1,k} \leq (1 - \delta_{\mathrm{a},k}), \tag{4.47m}$$
$$z_k + (1 - \delta_{\mathrm{a},k}) \leq 1 + \delta_{1,k}, \tag{4.47n}$$
$$\delta_{2,k} \leq (1 - z_k), \tag{4.47o}$$
$$\delta_{2,k} \leq \delta_{\mathrm{b},k}, \tag{4.47p}$$
$$(1 - z_k) + \delta_{\mathrm{b},k} \leq 1 + \delta_{2,k}, \tag{4.47q}$$
$$\delta_{3,k} \geq \delta_{1,k}, \tag{4.47r}$$
$$\delta_{3,k} \geq \delta_{2,k}, \tag{4.47s}$$
$$\delta_{3,k} \leq \delta_{1,k} + \delta_{2,k}, \tag{4.47t}$$
$$z_{k+1} = \delta_{3,k}. \tag{4.47u}$$

Here, the decision variables are x_k, w_k, u_k, all of which are continuous, along with binary variables $\delta_{\mathrm{a},k}$, $\delta_{\mathrm{b},k}$, $\delta_{1,k}$, $\delta_{2,k}$, $\delta_{3,k}$, and z_k. The total number of optimization variables amounts to $5N$. We remind that the index k goes from 0 to $N-1$, and constraints (4.47d)-(4.47g) are utilized for $i \in 1, \ldots, n_{\mathrm{u}}$. It is important to note that all constraints in (4.47) are linear in the decision variables. If we set the objective function similar to (3.14), then the entire problem becomes a mixed-integer quadratic problem with linear constraints. The problem is initialized by the state measurements $x_0 = x(t)$, and by the current state of the relay-based controller $z_0 = z(t)$. As a final remark, the prediction equation in (4.47b) and (4.47c) can be extended by with accordance to offset-free modeling (Section 3.2) in order to compensate for the effects of the unmeasured disturbances. Let us again emphasize, that a zero offset tracking is hardly possible due to the binary nature of the manipulated variable u.

The application of the MPC-based reference governor presented in this section is addressed in the Chapter 7, where a simulation-based case study involving a thermostatically controlled temperature in building in covered.

4.3 Systems with Inner MPC Controllers

The objective of this section is to present a systematic and computationally tractable way of implementing reference governors where the inner feedback loops contain individual MPC controller to control corresponding subsystems. These controllers, however, are not aware of coupling constraints that are only known to the reference governor. In this chapter we review two approaches how to deal with the reference governor formulations The first approach (the Section 4.3.2) uses the Karush-Kuhn-Tucker (KKT) conditions of the inner controllers. Although these conditions are nonlinear, we show that they can be rewritten as a set of convex constraints that involve binary variables. Although the resulting problem is still non-convex, it can be solved to global optimality using off-the-shelf mixed-integer solvers. The second approach, discussed in the Section 4.3.3, is based on pre-computing, off-line, the explicit solution for the inner MPC controllers. This solution can, again, be encoded using binary variables. Findings in this chapter are adopted from the paper by Holaza, Klaučo, and Kvasnica [2017].

4.3.1 Local MPC and MPC-based Reference Governors

Consider a set of M linear time invariant models (3.12) denoted as

$$x_i(t+T_\mathrm{s}) = A_i x_i(t) + B_i u_i(t), \tag{4.48a}$$

$$y_i(t) = C_i x_i(t) + D_i u_i(t), \tag{4.48b}$$

where $x_i \in \mathbb{R}^{n_\mathrm{x}}$, $u_i \in \mathbb{R}^{n_\mathrm{u}}$, $y_i \in \mathbb{R}^{n_\mathrm{y}}$ are the state, input and output vectors of the i-th subsystem, respectively. Without loss of generality we will assume that the dimensions of all systems are identical. Each subsystem must be operated such that its local constraints are satisfied:

$$x_i \in \mathcal{X}_i,\ u_i \in \mathcal{U}_i,\ y_i \in \mathcal{Y}_i, \tag{4.49}$$

where $\mathcal{X}_i \subseteq \mathbb{R}^{n_\mathrm{x}}$, $\mathcal{U}_i \subseteq \mathbb{R}^{n_\mathrm{u}}$, $\mathcal{Y}_i \subseteq \mathbb{R}^{n_\mathrm{y}}$ are polyhedral sets, and $\forall i \in \mathbb{N}_1^M$. Moreover, the individual subsystems are assumed to be coupled by coupling con-

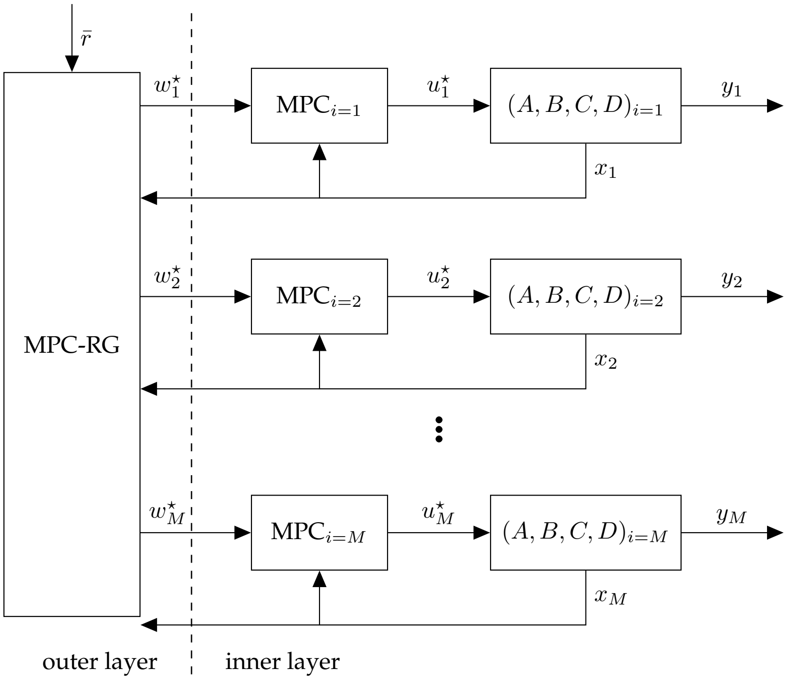


Figure 4.7: Block diagram of the considered reference governor setup with inner MPC controllers controlling individual subsystems.

straints

$$\begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} \in \overline{\mathcal{X}}, \quad \begin{bmatrix} u_1 \\ \vdots \\ u_M \end{bmatrix} \in \overline{\mathcal{U}}, \quad \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} \in \overline{\mathcal{Y}}, \tag{4.50}$$

where

$$\overline{\mathcal{X}} \subseteq (\mathcal{X}_1 \times \cdots \times \mathcal{X}_M) \subseteq \mathbb{R}^{Mn_\mathrm{x}} \tag{4.51a}$$
$$\overline{\mathcal{U}} \subseteq (\mathcal{U}_1 \times \cdots \times \mathcal{U}_M) \subseteq \mathbb{R}^{Mn_\mathrm{u}} \tag{4.51b}$$
$$\overline{\mathcal{Y}} \subseteq (\mathcal{Y}_1 \times \cdots \times \mathcal{Y}_M) \subseteq \mathbb{R}^{Mn_\mathrm{y}}. \tag{4.51c}$$

The overall arrangement is depicted on the Fig. 4.7, where each subsystem in (4.48) is individually controlled by its own MPC feedback policy in the form

$$u_i^\star(t) = \mathrm{MPC}_i(\theta_i(t)), \tag{4.52}$$

where $\theta_i(t)$ are the initial conditions of the i-th controller at the time instant t, such as the state measurements $x_i(t)$, output references, etc. The MPC controllers are assumed to be designed such that they enforce satisfaction of state, input and output constraints in (4.49), but they *do not* have any knowledge of the coupling constraints in (4.50), neither they communicate.

The aim is to design an MPC-based reference governor that shapes the individual references w_i for the separate inner loops MPC controller $\text{MPC}_i(\theta_i(t))$ in such a way, that

1. the coupling constraints in (4.50) are enforced,
2. the outputs of the individual subsystems track the user-specified references $\bar{r} = (r_1, \ldots, r_M)$ as closely as possible.

The local MPC controller $u_i^\star(t) = \text{MPC}_i(\theta_i(t))$ is implemented in the receding horizon fashion, hence the

$$u_i^\star(t) = \Phi U_i(\theta_i(t)), \tag{4.53}$$

where

$$\Phi = [I_{n_\mathrm{u}} \ \underbrace{0_{n_\mathrm{u}\times n_\mathrm{u}} \cdots \ 0_{n_\mathrm{u}\times n_\mathrm{u}}}_{N_i-1}], \tag{4.54}$$

with N_i being the prediction horizon for the MPC controlling the i-th subsystem. Particularly, the local MPC is casted as

$$\min \sum_{k=0}^{N_i-1} \left(||y_{i,k} - w_i||^2_{Q_{\mathrm{y},i}} + ||u_{i,k} - u_{i,k-1}||^2_{Q_{\mathrm{u},i}} \right) \tag{4.55a}$$

$$\text{s.t.} \ x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} \tag{4.55b}$$

$$y_{i,k} = C_i x_{i,k} + D_i u_{i,k}, \tag{4.55c}$$

$$x_{i,k} \in \mathcal{X}_i, \tag{4.55d}$$

$$u_{i,k} \in \mathcal{U}_i, \tag{4.55e}$$

$$y_{i,k} \in \mathcal{Y}_i, \tag{4.55f}$$

$$x_{i,0} = x_i(t), \ u_{i,-1} = u_i(t - T_\mathrm{s}) \tag{4.55g}$$

where constraints (4.55b)–(4.55f) are enforced for $k = 0, \ldots, N_i - 1$ with N_i. The vector of initial parameters contains the initial conditions as in (4.55g) and also the sequence of the references w_i. Formally, we can write

$$\theta_i = \begin{bmatrix} x_{i,0}^\intercal & w_i^\intercal & u_{i,-1}^\intercal \end{bmatrix}^\intercal. \tag{4.56}$$

The optimization is performed with respect to the sequence of optimal control input U_i, which is given as

$$U_i = \begin{bmatrix} u_{i,0}^\mathsf{T} & \dots & u_{i,N_i-1}^\mathsf{T} \end{bmatrix}^\mathsf{T}, \tag{4.57}$$

furthermore, all of the weighting factors $Q_{\mathrm{y},i}$, and $Q_{\mathrm{u},i}$ are positive definite matrices, hence the problem (4.55) is strictly a convex optimization problem.

REMARK 4.5. Note, that the control problem in (4.55) can be extended with the future value of the references, so the first term in the objective function will change to

$$||Q_{\mathrm{y},i}\left(y_{i,k} - w_{i,k}\right)||_2^2. \tag{4.58}$$

Next, the model in (4.55b) and (4.55c) can be extended by disturbances with accordance to the Section 3.2. Such a modification to the local MPC problem will result in

$$\theta_i = \begin{bmatrix} x_{i,0}^\mathsf{T} & w_{i,1}^\mathsf{T} & \dots & w_{i,(N_i-1)}^\mathsf{T} & u_{i,-1}^\mathsf{T} & d_{i,0}^\mathsf{T} \end{bmatrix}^\mathsf{T}, \tag{4.59}$$

which is an extended vector of initial conditions. □

The role of the MPC-based reference governor is to determine the at each sampling instant t an optimal sequence of shaped references $w_i(t)$ for $i = 1, \dots, M$ for the local MPC problems (4.55) such that

1. the tracking error $||r_i - y_i||$ is minimized,
2. and the coupling constraints (4.50) are enforced.

The MPC-based reference governor is then casted as

$$\min \sum_{j=0}^{\overline{N}} \sum_{i=1}^{M} \left(\|y_{i,j} - r_i\|_{Q_{\mathrm{y},i}}^2 + \|w_{i,j} - w_{i,j-1}\|_{Q_{\mathrm{w},i}}^2 \right), \tag{4.60a}$$

$$\text{s.t. } U_{i,j}^\star(\theta_{i,j}) = \arg\min \text{ (4.55a) s.t. (4.55b)} - \text{(4.55f)}, \tag{4.60b}$$

$$u_{i,j}^\star = \Phi U_{i,j}^\star(\theta_{i,j}), \tag{4.60c}$$

$$x_{i,j+1} = A_i x_{i,j} + B_i u_{i,j}^\star, \tag{4.60d}$$

$$y_{i,j} = C_i x_{i,j} + D_i u_{i,j}^\star, \tag{4.60e}$$

$$\bar{x}_j \in \overline{\mathcal{X}}, \tag{4.60f}$$

$$\bar{u}_j^\star \in \overline{\mathcal{U}}, \tag{4.60g}$$

$$\bar{y}_j \in \overline{\mathcal{Y}}, \tag{4.60h}$$

with (4.60b)−(4.60h) imposed for $t = 0, \ldots, \overline{N} - 1$, and (4.60b)−(4.60e) also for $i = 1, \ldots, M$, where $\overline{N}$ is the prediction horizon of the outer MPC problem. The reference governor problem in (4.60) is coupled with individual MPC controllers via the optimally shaped references w_i that enter (4.55) through $\theta_{i,j}$ in (4.60b) and (4.60c). The vector of initial conditions for the local MPC at the j-th time step is defined as

$$\theta_{i,j} = \begin{bmatrix} x_{i,j}^{\intercal} & w_{i,j}^{\intercal} & u_{i,j-1}^{\intercal} \end{bmatrix}^{\intercal}. \tag{4.61}$$

The variables in the coupling constraints in (4.60f)-(4.60h) are defined as follows

$$\bar{x}_j = \begin{bmatrix} x_{1,j}^{\intercal} & \ldots, x_{M,j}^{\intercal} \end{bmatrix}^{\intercal}, \tag{4.62a}$$

$$\bar{u}_j = \begin{bmatrix} u_{1,j}^{\intercal} & \ldots, u_{M,j}^{\intercal} \end{bmatrix}^{\intercal}, \tag{4.62b}$$

$$\bar{y}_j = \begin{bmatrix} y_{1,j}^{\intercal} & \ldots, y_{M,j}^{\intercal} \end{bmatrix}^{\intercal}, \tag{4.62c}$$

and the vector of optimal control sequences $U_{i,j}^{\star}$ reads to

$$U_{i,j}^{\star} = \begin{bmatrix} {u_{i,j}^{\star}}^{\intercal} & \ldots & {u_{i,j+N_i-1}^{\star}}^{\intercal} \end{bmatrix}^{\intercal}. \tag{4.63}$$

Lastly, in the MPC-based reference governor design, we allow for the prediction horizon of the governor to be different from the horizons of local MPCs.

The closed-loop implementation of the MPC-based reference governor (cf. Fig. 4.7) is done in the tradition receding horizon fashion:

1. Measure (or estimate) states $x_i(t)$ of the individual subsystems;
2. Initialize (4.60) by $x_{i,0} = x_i(t)$, $u_{i,-1}^{\star} = u_i^{\star}(t - T_{\mathrm{s}})$, r_i (the user-specified references), $w_{i,-1}^{\star} = w_i^{\star}(t - T_{\mathrm{s}})$ for $i = 1, \ldots, M$ and solve (4.60) for $w_{i,0}^{\star}$ (the optimally shaped references);
3. Feed $w_{i,0}^{\star}$ as setpoints to the inner MPC controllers;
4. Repeat from Step 1 at the subsequent time instant.

When the procedure is commenced at $t = 0$, any $u_{i,-1} \in \mathcal{U}_i$ can be chosen as the initial value of the "previous" control action, together with $w_{i,-1}^{\star} = r_i$.

The difficulty of solving for the shaped references $w_i^{\star}$, $i = 1, \ldots, M$ from (4.60) stems from the fact that the control sequences $U_{i,t}^{\star}$ in (4.60b) are given as the solution of inner optimization problems (4.55). The problem in (4.60) is thus a *bilevel* optimization problem, which is non-convex.

4.3.2 KKT Conditions Reformulation

In this section, we show how the non-convex bilevel optimization problem (4.60) can be rewritten by reformulating the inner optimization problem in (4.60b) via its KKT conditions. Consider, that the inner MPC problem can be rewritten as a parametric quadratic problem as stated in (2.8), hence

$$\min_{U_i} \ \frac{1}{2} U_i^\intercal H_i U_i + \theta_i^\intercal F_i U_i + c_i^\intercal U_i \tag{4.64a}$$
$$\text{s.t.} \ \ G_i U_i \leq V_i + E_i \theta_i, \tag{4.64b}$$

with U_i being defined per (4.57).

To ease the notation, henceforth we will consider a particular subsystem (4.55) and we will omit the index i. Since the QP in (4.64) is strictly convex, $U^\star$ is its minimizer if and only if the primal-dual pair $(U^\star, \lambda^\star)$ satisfies the Karush-Kuhn-Tucker (KKT) conditions

$$HU^\star + F^\intercal \theta + c + G^\intercal \lambda^\star = 0, \tag{4.65a}$$
$$GU^\star - V - E\theta \leq 0, \tag{4.65b}$$
$$\lambda^\star \geq 0, \tag{4.65c}$$
$$\lambda_s^\star (G^{[s]} U^\star - V^{[s]} - E^{[s]}\theta) = 0, \quad \forall s \in \mathbb{N}_1^{n_\mathrm{G}}, \tag{4.65d}$$

where $G^{[s]}$ denotes the s-th row of the corresponding matrix, n_G is the number of constraints in (4.64b), and $\lambda \in \mathbb{R}^{n_\mathrm{G}}$ is the vector of Lagrange multipliers.

Notice that the KKT conditions (4.65) are nonlinear due to the product between λ and U in the complementarity slackness condition (4.65d). One could straightforwardly replace $U_{i,t}^\star(\theta)$ as the optimal solution to the inner problem in (4.60b) by the corresponding KKT conditions (4.65), but then the reference governor problem (4.60) would become a nonlinear programming problem (NLP) that is difficult to solve to global optimality. Therefore we show how to formulate (4.65) as a set of linear constraints that involve binary decision variables. Although the problem will still be non-convex, it can be solved to global optimality using off-the-shelf mixed-integer solvers, such as CPLEX, GUROBI, or MOSEK.

Observe that for (4.65d) to be satisfied, whenever the s-th constraint is inactive, the corresponding Lagrange multiplier λ_s must be zero at the optimum, i.e.,

$$[G^{[s]} U^\star - V^{[s]} - E^{[s]}\theta < 0] \implies [\lambda_s^\star = 0]. \tag{4.66}$$

To model this logic relation, introduce binary variables $\delta_s \in \{0,\ 1\}$, $s = 1, \ldots, n_\mathrm{G}$, and rewrite (4.66) into

$$
\begin{aligned}
[G^{[s]}U^\star - V^{[s]} - E^{[s]}\theta < 0] &\implies [\delta_s = 1], && \text{(4.67a)}\\
[\delta_s = 1] &\implies [\lambda_s^\star = 0]. && \text{(4.67b)}
\end{aligned}
$$

Logical rules presented in (4.67) can be rewritten to a set of linear inequalities. The procedure was already described in the Section 4.2.3 and in Lemma 4.1, but specifically, consider function $f(z),\ z \in \mathbb{R}^{n_z}$ and an arbitrary binary variable σ. Then:

1. $[f(z) < 0] \implies [\sigma = 1]$ if and only if $f(z) \geq Z_{\text{min}}\sigma$,
2. $[\sigma = 1] \implies [f(z) = 0]$ if and only if $Z_{\text{min}}(1-\sigma) \leq f(z) \leq Z_{\text{max}}(1-\sigma)$,
3. $[\sigma = 1] \implies [f(z) \leq 0]$ if and only if $f(z) \leq Z_{\text{max}}(1-\sigma)$,

with $Z_{\text{min}} \leq \min f(z)$ and $Z_{\text{max}} \geq \max f(z)$.

Applying the procedure to the implication rules in (4.67), we arrive at

$$
\begin{aligned}
&G^{[s]}U^\star - V^{[s]} - E^{[s]}\theta \geq Z_{\text{min},s}\delta_s, && \text{(4.68a)}\\
&Z_{\text{min},s}(1-\delta_s) \leq \lambda_s^\star \leq Z_{\text{max},s}(1-\delta_s), && \text{(4.68b)}
\end{aligned}
$$

which for all $s = 1, \dots, n_{\text{G}}$ are linear inequalities in $U^\star$, $\lambda^\star$, and δ_s. Consider

$$
\begin{aligned}
\lambda &= \begin{bmatrix} \lambda_1 & \dots & \lambda_{n_{\text{G}}} \end{bmatrix}^\intercal, && \text{(4.69a)}\\
\delta &= \begin{bmatrix} \delta_1 & \dots & \delta_{n_{\text{G}}} \end{bmatrix}^\intercal, && \text{(4.69b)}
\end{aligned}
$$

then the mixed-integer formulation of the KKT conditions in (4.65) becomes

$$
\begin{aligned}
HU^\star + F^\intercal\theta + c + G^\intercal\lambda^\star &= 0, && \text{(4.70a)}\\
GU^\star - V - E\theta &\leq 0, && \text{(4.70b)}\\
\lambda^\star &\geq 0, && \text{(4.70c)}\\
GU^\star - V - E\theta &\geq Z_{\text{min}}\delta, && \text{(4.70d)}\\
\lambda^\star &\geq Z_{\text{min}}(\mathbf{1}_{n_{\text{G}}} - \delta), && \text{(4.70e)}\\
\lambda^\star &\leq Z_{\text{max}}(\mathbf{1}_{n_{\text{G}}} - \delta). && \text{(4.70f)}
\end{aligned}
$$

Notice that (4.70e) is in fact redundant due to (4.70c) and can therefore be omitted.

Then the MPC-based reference governor problem in (4.60) can be equivalently reformulated by replacing (4.60b) with (4.70), imposed for all $j = 0, \dots, \overline{N}$

(i.e., for each step of the prediction horizon of the reference governor) and $i = 1, \dots, M$ (i.e., for each subsystem):

$$
\begin{aligned}
\min_{\lambda_{i,j},\delta_{i,j},u_{i,j},w_{i,j}} \quad & \sum_{j=0}^{\overline{N}-1} \sum_{i=1}^{M} \left(\|y_{i,j} - r_i\|_{Q_{\mathrm{y},i}}^2 + \|w_{i,j} - w_{i,j-1}\|_{Q_{\mathrm{w},i}}^2 \right), && \text{(4.71a)} \\
\text{s.t.} \quad & H_i U_{i,j}^\star + F^\intercal \theta_{i,t} + c_i + G_i^\intercal \lambda_{i,j} = 0, && \text{(4.71b)} \\
& G_i U_{i,j}^\star - V_i - E_i \theta_{i,j} \leq 0, && \text{(4.71c)} \\
& \lambda_{i,j} \geq 0, && \text{(4.71d)} \\
& G_i U_{i,j}^\star - V_i - E_i \theta_{i,j} \geq Z_{\min} \delta_{i,j}, && \text{(4.71e)} \\
& \lambda_{i,j} \leq Z_{\max}(\mathbf{1}_{n_{\mathrm{G}}} - \delta_{i,j}), && \text{(4.71f)} \\
& u_{i,j}^\star = \Phi U_{i,j}^\star, && \text{(4.71g)} \\
& x_{i,j+1} = A_i x_{i,t} + B_i u_{i,j}^\star, && \text{(4.71h)} \\
& y_{i,j} = C_i x_{i,j} + D_i u_{i,j}^\star, && \text{(4.71i)} \\
& \bar{x}_j \in \overline{\mathcal{X}}, && \text{(4.71j)} \\
& \bar{u}_j^\star \in \overline{\mathcal{U}}, && \text{(4.71k)} \\
& \bar{y}_j \in \overline{\mathcal{Y}}, && \text{(4.71l)} \\
& \delta_{i,j} \in \{0,1\}^{n_{\mathrm{G},i}}. && \text{(4.71m)}
\end{aligned}
$$

Since all constraints in (4.71) are linear, and the objective function is quadratic, the reference governor problem can be formulated as a mixed-integer quadratic program (MIQP). The total number of binary variables is $\overline{N} \sum_{i=1}^{M} n_{\mathrm{G},i}$.

The key limitation of solving the MIQP (4.71) is the number of binary variables. To reduce this number, one can remove redundant inequalities from the constraints in (4.64b) by solving, off-line, one linear program for each constraint, cf. [Bemporad, 2015].

4.3.3 Analytic Reformulation

An another computationally tractable way of solving the non-convex bilevel optimization problem in (4.60) is to replace (4.60b) by the analytical solution to the inner optimization problems (4.55). Specifically, consider (4.55) transformed into the equivalent form (4.64). When θ_i in (4.64) is considered as a free parameter, the problem is a strictly convex parametric QP (pQP) that since $H \succ 0$ as elaborated above. The analytical solution is in the form of a PWA function (2.9). In case of multiple local MPCs, we will obtain as many PWA functions. Specifically

$$\kappa_i(\theta_i) = \begin{cases} \alpha_{i,1}\theta_i + \beta_{i,1} & \text{if } \theta_i \in \mathcal{R}_{i,1} \\ & \vdots \\ \alpha_{i,n_{\mathrm{R},i}}\theta_i + \beta_{i,n_{\mathrm{R},i}} & \text{if } \theta_i \in \mathcal{R}_{i,n_{\mathrm{R},i}}, \end{cases} \tag{4.72}$$

with $n_{\mathrm{R},i}$ denoting the number of polyhedral regions for the i-th system. For more details about the explicit MPC scheme, refer to the Section 3.3 and literature therein.

Once the closed-form solutions $U_i^\star = \kappa_i(\theta_i)$ are determined for each subsystem $i = 1, \ldots, M$, (4.60b) can be replaced by (4.72) in the following way. For each critical region of the i-th closed-form solution, introduce binary variables $\delta_{i,1}, \ldots, \delta_{i,n_{\mathrm{R},i}}$. Then the inclusion $\theta_i \in \mathcal{R}_{i,r}$ is modeled by

$$[\delta_{i,r} = 1] \implies [\Gamma_{i,r}\theta_i \leq \gamma_{i,r}]. \tag{4.73}$$

Since the map κ_i is continuous and $\cup_r \mathcal{R}_{i,r}$ is a convex set, the constraint

$$\sum_{r=1}^{n_{\mathrm{R},i}} \delta_{i,r} = 1 \tag{4.74}$$

enforces that, if $\theta_i \in \mathcal{R}_{i,r}$, then $\delta_{i,r} = 1$ and the remaining variables will be zero, i.e., $\delta_{i,q} = 0\ \forall q \neq r$. Then

$$[\delta_{i,r} = 1] \implies [U_i^\star = \alpha_{i,r}\theta_i + \beta_{i,r}] \tag{4.75}$$

is optimal in (4.55). Applying the second and the third statement of Lemma 4.1 to (4.73) and (4.75) transforms these logic relations into linear inequalities involving continuous variables θ_i, $U_i^\star$ and binary variables $\delta_i = (\delta_{i,1}, \ldots, \delta_{i,n_{\mathrm{R},i}})$:

$$\Gamma_{i,r}\theta_i - \gamma_{i,r} \leq Z_{\max}(1 - \delta_{i,r}), \tag{4.76a}$$

$$Z_{\min}(1 - \delta_{i,r}) \leq U_i^\star - \alpha_{i,r}\theta_i - \beta_{i,r} \leq Z_{\max}(1 - \delta_{i,r}), \tag{4.76b}$$

where $Z_{\min}$ and $Z_{\max}$ are sufficiently small/large constants, cf. Lemma 4.1 or [Williams, 1993].

The inner optimization problems in (4.60b) can therefore be equivalently replaced by (4.76) together with (4.74). Then the reference governor problem in (4.60) becomes

$$\min_{\delta_{i,j,r}, w_{i,j}} \sum_{j=0}^{\overline{N}-1} \sum_{i=1}^{M} \left(\|y_{i,j} - r_i\|^2_{Q_{\mathrm{y},i}} + \|w_{i,j} - w_{i,j-1}\|^2_{Q_{\mathrm{w},i}} \right), \tag{4.77a}$$

$$\text{s.t. } \Gamma_{i,r}\theta_{i,j} - \gamma_{i,r} \leq Z_{\max}(1 - \delta_{i,j,r}), \tag{4.77b}$$

$$U^{\star}_{i,j} - \alpha_{i,r}\theta_{i,j} - \beta_{i,r} \geq Z_{\min}(1 - \delta_{i,j,r}), \tag{4.77c}$$

$$U^{\star}_{i,j} - \alpha_{i,r}\theta_{i,j} - \beta_{i,r} \leq Z_{\max}(1 - \delta_{i,j,r}), \tag{4.77d}$$

$$\mathbf{1}^{\intercal}_{n_{\mathrm{R},i}}\delta_{i,j} = 1, \tag{4.77e}$$

$$u^{\star}_{i,j} = \Phi U^{\star}_{i,j}, \tag{4.77f}$$

$$x_{i,j+1} = A_i x_{i,j} + B_i u^{\star}_{i,j}, \tag{4.77g}$$

$$y_{i,j} = C_i x_{i,j} + D_i u^{\star}_{i,j}, \tag{4.77h}$$

$$\bar{x}_t \in \overline{\mathcal{X}}, \tag{4.77i}$$

$$\bar{u}^{\star}_t \in \overline{\mathcal{U}}, \tag{4.77j}$$

$$\bar{y}_t \in \overline{\mathcal{Y}}, \tag{4.77k}$$

$$\delta_{i,j,r} \in \{0, 1\}, \tag{4.77l}$$

with (4.77b)–(4.77e) imposed for $i = 1, \ldots, M$, $j = 0, \ldots, \overline{N} - 1$, and (4.77b)–(4.77d) also for $r = 1, \ldots, n_{\mathrm{R},i}$. The optimization variables in this case are the shaped references $w_{i,j}$ and binary variables $\delta_{i,j,r}$. Recall, that the optimally shaped reference w enters the local MPC via the vector of initial conditions θ_i per (4.61). Contrary to the MIQP formulation (4.71) in the previous section, where also the vector of manipulated variables $U^{\star}_i$ was also optimized, here this variables is determined explicitly solely based on the choice of $\delta_{i,j,r}$. Nect, since all constraints in (4.74), (4.76), as well as in (4.60b)–(4.60h) are linear and the objective function (4.60a) is quadratic, the augmented problem (4.77) is, again, a mixed-integer QP. Again, the key limitation of the MIQP is the number of binary variables, which in this case it is determined by expression $\overline{N}\sum_{i=1}^{M} n_{\mathrm{R},i}$.

Various complexity reduction techniques can be used to reduce the number of critical regions in the explicit maps (4.72). For instance, one can resort to optimal region merging [Geyer et al., 2008], or to clipped PWA functions [Kvasnica and Fikar, 2012] to reduce R_i and thus the number of binary variables in (4.77). Alternatively, one can use approximate replacements of κ_i in (4.72) as in Holaza et al. [2015] at the expense of suboptimal performance of (4.77).

4.3.4 Example

Lets consider following example to show the performance of the MPC-based reference governor supervising local MPC strategies. Specifically, we consider

$M = 3$ subsystems that represent movement of frictionless vehicles, modeled as a double integrator with a variable mass with $T_\mathrm{s} = 1$:

$$x_i(t + T_\mathrm{s}) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_i(t) + \begin{bmatrix} 1/m_i \\ 0.5/m_i \end{bmatrix} u_i(t), \tag{4.78a}$$

$$y_i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x_i(t), \tag{4.78b}$$

where m_i is the mass of the i-th vehicle ($m_1 = 1.0$, $m_2 = 5.0$ and $m_3 = 0.5$, representing, e.g., a passanger car, a truck, and a motorcycle, respectively), $x_i \in \mathbb{R}^2$ consists of the vehicle's position and speed, u_i represents the manipulated acceleration, and the speed of the individual vehicle is the controlled output. Each vehicle is individually controlled by its own MPC controller, represented by (4.55) with $N_i = 4$, $Q_{\mathrm{y},i} = 10$, $Q_{\mathrm{u},i} = 1$, and

$$\mathcal{X}_i = \{x_i \in \mathbb{R}^2 \mid x_\mathrm{min} \leq x_i \leq x_\mathrm{max}\}, \tag{4.79a}$$

$$\mathcal{U}_i = \{u_i \in \mathbb{R} \mid -1 \leq u_i \leq 1\}, \tag{4.79b}$$

$$\mathcal{Y}_i = \{y_i \in \mathbb{R} \mid -1 \leq y_i \leq 1\} \tag{4.79c}$$

where

$$x_\mathrm{min} = \begin{bmatrix} -15 \\ 1 \end{bmatrix} \quad x_\mathrm{max} = \begin{bmatrix} 50 \\ 1 \end{bmatrix} \tag{4.80}$$

In addition, the individual subsystems are subject to coupling constraints that are unknown to inner-loop MPCs. Specifically, we require the vehicles not to collide. This constraint is translated to

$$\overline{\mathcal{X}} = \{(x_1, x_2, x_3) \in \mathbb{R}^6 \mid C_\mathrm{x}x_1 - C_\mathrm{x}x_2 \geq \epsilon,\ C_\mathrm{x}x_2 - C_\mathrm{x}x_3 \geq \epsilon\}, \tag{4.81}$$

where ϵ represents the minimal separation gap between vehicles with $\epsilon = 5$ in our simulations, and $C_\mathrm{x} = [1\ 0]$. Moreover, we assume

$$\overline{\mathcal{Y}} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_3 \tag{4.82a}$$

$$\overline{\mathcal{U}} = \mathcal{U}_1 \times \mathcal{U}_2 \times \mathcal{U}_3 \tag{4.82b}$$

The vehicles' starting positions are assumed to be such that $p_1(0) \geq p_2(0) \geq p_3(0)$ with $p_i = C_\mathrm{x}x_i$.

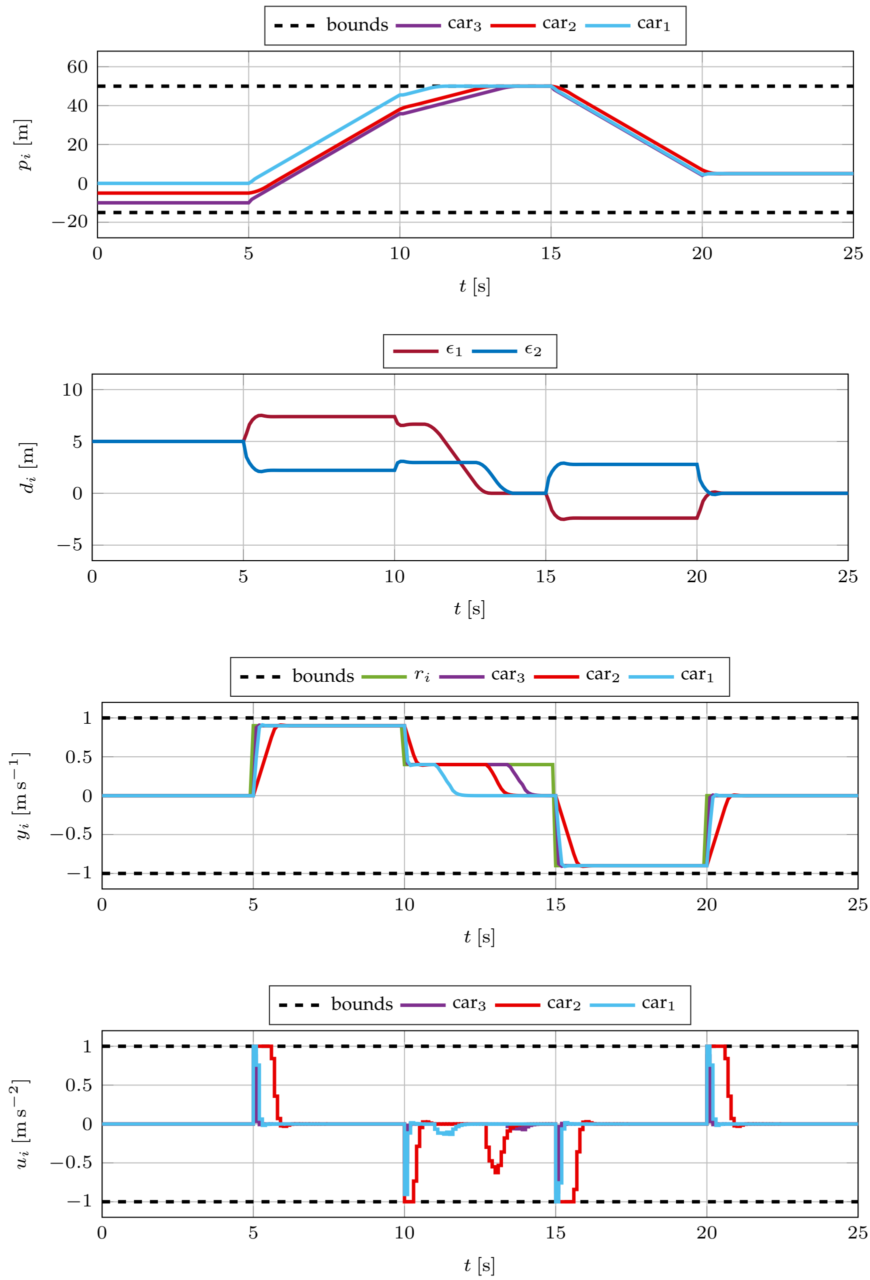


Figure 4.8: Closed-loop simulation results for the scenario with no reference governor. The plots depict, respectively, the positions of individual vehicles p_i, the separation gap d_i between them (required to be ≥ 5 for no collision), the controlled speed y_i, and the respective manipulated inputs u_i.

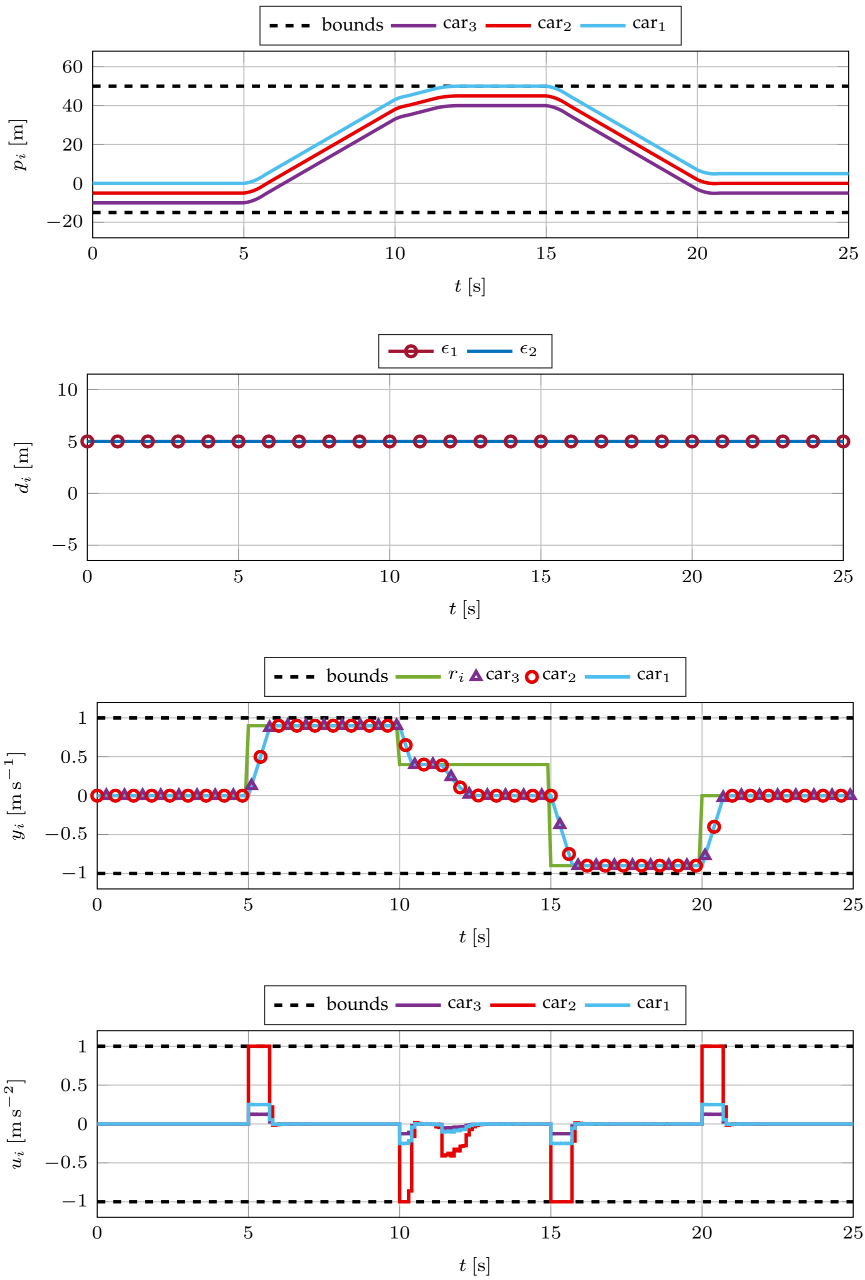


Figure 4.9: Closed-loop simulation results for the scenario with reference governor inserted into the loop. The plots depict, respectively, the positions of individual vehicles p_i, the separation gap d_i between them (required to be ≥ 5 for no collision), the controlled speed y_i, and the respective manipulated inputs u_i.

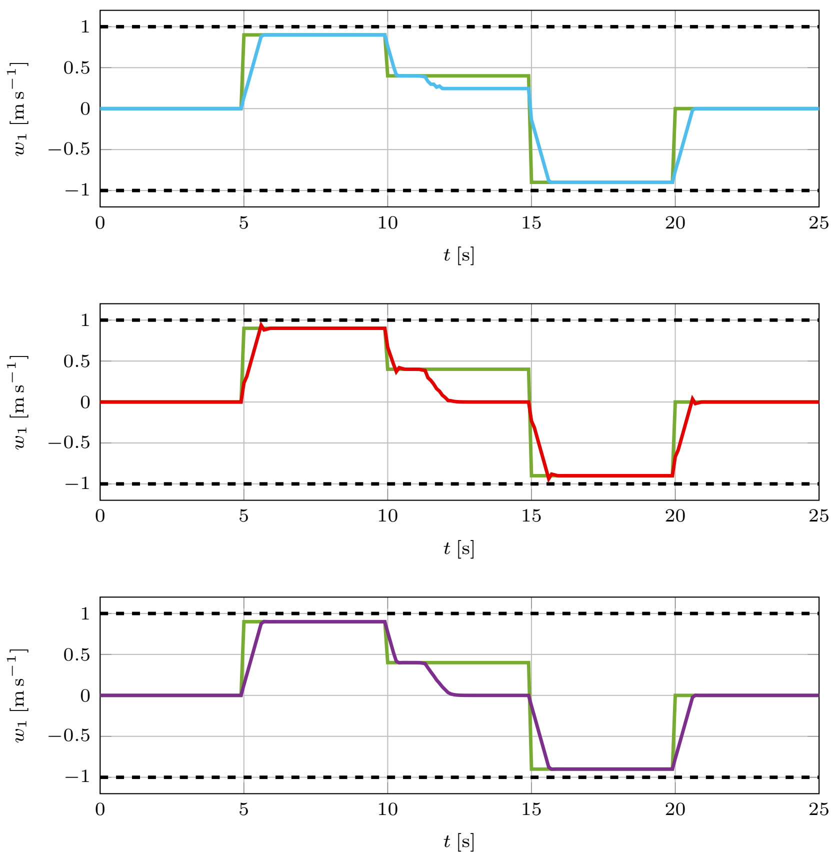


Figure 4.10: References $w_i^\star$ optimally shaped by the reference governor.

First, to illustrate the need for a coordinating reference governor, we have performed a closed-loop simulation starting from

$$x_1(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x_3(0) = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \tag{4.83}$$

where the individual subsystems are controlled by their respective MPC feedback policies (4.55) in the absence of coupling constraints. Since no reference governor is assumed in this scenario, $w_i = r_i$ in (4.55a). The results for a time-varying profile of the velocity trajectories r_i to be tracked are shown in Fig. 4.8. As can be observed, although the inner controllers manipulate the control inputs such that the velocity references are tracked, they violate (cf. the second subplot in Fig. 4.8) the collision-free coupling constraints in (4.81) as they have no knowledge about them. Notice that, although inner MPC loops maintain constraints of individual subsystems in (4.79), in absence of the reference governor they violate the coupling constraints in (4.81) (notice that $d_i < 5$ in the second subplot) starting from the 5th simulation step and never recover. The ϵ_i quantity denotes the gap between the i-th and $(i+1)$-th vehicle.

To enforce satisfaction of (4.81) we have therefore inserted the proposed reference governor (4.60) into the loop. It was designed for $\overline{N} = 4$, $Q_{\mathrm{y},i} = 10$, and $Q_{\mathrm{w},i} = 1$ for $i = 1, 2, 3$. The reference governor, represented by the bilevel problem (4.60), was subsequently formulated as a MIQP (4.71) per Section 4.3.2 using YALMIP [Löfberg, 2004] and solved by GUROBI. Specifically, the inner MPC problems, translated to (4.64), had $n_{R_\mathrm{i}} = 34$ for all subproblems, therefore (4.71) has 408 binary variables. The results of the closed-loop simulation starting from the same initial conditions as reported above are visualized in Fig. 4.9. Here, we can see from its second subplot that the minimal separation gap, represented by the coupling constraint (4.81), is maintained. The reference governor achieves this behavior by replacing the user-specified references r_i by optimized ones ($w_i^\star$), as shown in Fig. 4.10. Notice that the separation gap is kept above the safety margin ($d_i \geq 5$) at all time due to the reference governor enforcing (4.81) by suitably modifying the velocity references, cf. Fig. 4.10. Note how $w_i^\star$ start to diverge from user-specified setpoints r_i when such a change is required to enforce the coupling constraint in (4.81) during setpoint transitions, and also between $t = 10$ and $t = 15$. Once recovered from a dangerous situation, $w_i^\star$ converge to r_i again.The average per-sample runtime required to solve (4.71) was $0.28\,\mathrm{s}$ seconds with the maximum being $2.64\,\mathrm{s}$.

Next, we have compared the KKT-based formulation in (4.71) to the approach of Section 4.3.3 based on an analytic solution to (4.55). To do so, we have first computed the closed-form solutions κ_i as in (4.72) by solving the inner MPC problems (4.52) using MPT toolbox [Herceg et al., 2013]. Since each subsystem assumes a different B matrix in (4.78), explicit solutions with a different number of regions were obtained. Specifically, we got $R_1 = 107$ regions for the first subsystem's MPC, $R_2 = 127$ for the second, and $R_3 = 89$ for the third. These were obtained off-line in $21.8\,\mathrm{s}$. Consequently, the reference governor formulation in (4.77) had 1292 binary variables. The average per-sample runtime required to solve (4.77) at each step of the simulation was $7.9\,\mathrm{s}$ with the maximum being $117.3\,\mathrm{s}$. Needless to say, as both (4.71), as well as (4.77) are equivalent to (4.60), identical simulation results were obtained.

4.4 Concluding Remarks

This chapter discussed the theoretical basis of the MPC-based reference governors. We have shown, how the MPC-based governor is structured for three main choices of the primary controllers.

First, we show the formulation of the governor for the closed-loop system, which consists of PID controllers (the section 4.1). In here, we have modeled the closed-loop system consisting of a several PID controllers into a one aggregated state space model. This state space model is then used as a prediction model for the MPC-based governor. The resulting optimization problem formulation boils down to a quadratic optimization problem, which can be tractably solved via GUROBI, MOSEK or other state-of-the-art solvers. In fact, the formulation of the MPC-based reference governor in, e.g., (4.20) is basically the same as a Direct-MPC formulation (3.13), discussed in the Section 3.1.2. The reason is, the aggregation of the state space models of the PID controllers and the plant itself. The application of this MPC-RG strategy is further discussed in two case studies, in chapters 5 and 6.

The second case discussed, was a closed-loop system which consists of an "on-off" controller. Compared to the previous MPC-RG strategy, here the cornerstone was to derive the model of the switching logic. This was carried out by employing propositional logic [Williams, 1993]. Here, the formulation of the MPC-based governor (4.47) resulted in a mixed-integer optimization problem. Since the objective function is chosen either linear or quadratic and the constraints are linear in decision variables, the problem can be again solved in tractable manner by professional tools, like GUROBI, MOSEK or CPLEX. Naturally, the number of binary variables in this case is the limiting factor of scalability. The viability of the MPC-based governor supervising the relay-based controllers is tested on a simulation-based case study with results in the chapter 7.

Lastly, in the section 4.3, we show how to tackle MPC-based reference governor setup for closed-loops consisting of local MPC controllers. The main challenge stems from the nature of the optimization problem (4.60), which is bilevel. This problem is non-convex in general, and can not be solved efficiently by available solvers. To mitigate this problem, we propose two reformulation options of this bilevel optimization problem. The first approach was based on replacing the inner MPC problems by its respective KKT conditions. The second approach assumes that the inner MPC is encoded as a polyhedral PWA function, i.e. solved explicitly. In both cases, we obtain a mixed-integer quadratic optimization problem. The tractability of the solution is entirely problem-dependent, since the variation in the number of binary variables, which is the main limiting factor, largely depends in the number of constraints of the inner problem etc. In this section, we offer a motivating example, to provide an illustration of how such a MPC-RG can be formulated and solved.

Part II

Case Studies and Applications

CHAPTER 5

Magnetic Levitation

In this chapter, we will illustrate the synthesis and experimental verification of the MPC-RG strategy as described in the Section 4.1.2. The MPC-RG formulation will be parametrically solved according to the Section 3.3 and implemented in real-time fashion on a micro-chip with limited computational and memory resources. Such an implementation require to encode the resulting parametric solution to a binary search tree as introduced in the Section 2.2.2. The viability of the MPC-RG strategy is tested on a laboratory process of a magnetically suspended ball.

5.1 Introduction to Real-Time Control Strategy

The choice of the explicit MPC stems from the control hardware we have available. Namely, it is an Atmel AT-Mega382p 8-bit microcontroller with 2kB of dynamic memory. A process involving a magnetically suspended ball is used to test the viability of the proposed explicit MPC-based governor. Such systems have a broad use in industrial applications where they serve to suspend materials in a magnetic field. For instance, it is already used in maglev trains [Lee et al., 2006], high-speed motors using magnetic bearings [Schuhmann et al., 2012], but also in some unusual applications such as 3D cell culturing [Haisler et al., 2013] or harvesting of kinetic energy from human movements [Berdy et al., 2015].

The scientific community offers several advanced control strategies focused on the stabilization and control of the magnetic levitation. We may start by Glück et al. [2011], a cascaded controller for self-sensing magnetic levitation along with the position estimation based on least squares identification, implemented on Altera Stratix II FPGA, is proposed. A wholly different approach to magnetic levitation control is provided in Bächle et al. [2013], where a computationally low-demand non-linear MPC is demonstrated on dSPACE platform. A robust

Table 5.1: Comparison to other related works.

	Bächle et al. [2013]	Folea et al. [2016]	Lin et al. [2014]	Klaučo et al. [2017]
Control scheme	Nonlinear online MPC	Fractional order contr.	FLCMAC	Explicit MPC
Constraints handling	input	none	none	input, output, states
Controller	dSPACE MicroAutoBox I 800 MHz	NI cRIO-9014 Embed. Contr. 400 MHz	Altera Stratix II FPGA	Atmel Sam3X Cortex-M3 84 MHz
Controller price [€]	thousands	thousands	hundreds	tens
Computation time	900μs	N/A ($<$ 2ms)	N/A ($<$ 100μs)	266μs
Sampling frequency	700 Hz	500 Hz	10 kHz	1 kHz

tracking control of magnetic levitation process with input constraints is proposed in Zhang et al. [2015]. Here the authors use the MATLAB-equipped PC to control the system. An interesting approach of magnetic levitation control design is provided in Folea et al. [2016], where a fraction order controller implemented in embedded microcontroller is used to increase the closed-loop performance and robustness. Very popular approach to the application of control algorithms for magnetic processes is the use of FPGA [Lin et al., 2011, 2014]. All of previously mentioned approaches has several shortcomings, like the price of the controller used or the structure of the control strategy, which does not allow to handle output constraints. A more detailed comparison is summarized in Table 5.1.

This chapter will present the parametric solution to the MPC-based reference governor strategy, which will account for both, the input and output constraints, and will run on a cheap hardware. Findings in this chapter are based on publications by [Kalúz, Klaučo, and Kvasnica, 2015] and [Klaučo, Kalúz, and Kvasnica, 2017].

5.2 Plant Description

This section describes the mathematical modeling of the magnetically suspended ball. Here, we also present the primary stabilizing PID controller and the state space model closed-loop system. Secondly, we introduce the laboratory device and experimental setup.

5.2.1 Mathematical Modeling

The mathematical model of magnetic levitation is derived from the standard physical understanding of magnetic suspension of an object. The spatial arrangement of the system is shown in Fig. 5.1, followed by a photo-capture of the experimental device. The arrangement consists of three main parts. These are a metal cylinder with a winding that makes an electromagnetic coil at the top, inductive proximity sensor at the base, and ferromagnetic ball in the space between them. The arrangement considers the displacement of the ball only in the vertical axis (denoted as p).

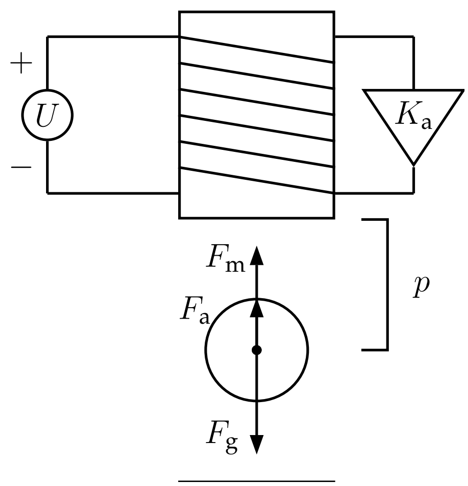


Figure 5.1: Magnetic levitation system.

The dynamics of the ball movement is derived from a standard force equation

$$F_\mathrm{a} = F_\mathrm{g} - F_\mathrm{m}, \tag{5.1}$$

where

$$F_\mathrm{a} = m_\mathrm{b} \frac{\mathrm{d}^2 p}{\mathrm{d}t^2}, \tag{5.2a}$$

$$F_\mathrm{g} = m_\mathrm{b} g, \tag{5.2b}$$

$$F_\mathrm{m} = K_\mathrm{m} \frac{I_\mathrm{c}^2}{p^2}. \tag{5.2c}$$

By the F_a we denote the vertical acceleration force, F_g is the gravitational force, and F_m is the force of the magnetic field acting in the ball. Next, m_b is the

Table 5.2: Parameters of the magnetic levitation plant.

Variable	Unit	Value
m_{b}	kg	$8.4 \cdot 10^{-3}$
g	$\mathrm{m\,s^{-2}}$	9.81
a_{ps}	$\mathrm{V\,m^{-1}}$	$-1.1132 \cdot 10^{3}$
b_{ps}	V	11.107
$U_{\mathrm{c}}^{\mathrm{s}}$	V	2.29
$U_{\mathrm{ps}}^{\mathrm{s}}$	V	1.40
K_{a}	$\mathrm{kg\,m^2\,s^{-2}\,V^{-1}}$	$1.1899 \cdot 10^{-6}$

mass of the ball, g is gravitational acceleration, K_{m} is magnetic constant, I_{c} is te electric current flowing through the coil.

Since the practical implementation relies on inputs and outputs to be in voltage units, the input voltage applied to the coil, i.e., U_{c}, can be expressed as

$$U_{\mathrm{c}} = I_{\mathrm{c}} R_{\mathrm{c}}. \tag{5.3}$$

Similarly, the output voltage taken from the position sensor, i.e., U_{ps}, can be expressed as a linear function of position

$$U_{\mathrm{ps}} = a_{\mathrm{ps}} p + b_{\mathrm{ps}}. \tag{5.4}$$

Using these two substitutions and introducing a conjured coil amplifier constant

$$K_{\mathrm{a}} = \frac{K_m}{R_c^2}, \tag{5.5}$$

hence, the model can be written as

$$\frac{m_{\mathrm{b}}}{a_{\mathrm{ps}}} \frac{d^2 U_{\mathrm{ps}}}{dt^2} = m_{\mathrm{b}} g - K_{\mathrm{a}} \frac{U_{\mathrm{c}}^2 a_{\mathrm{ps}}^2}{U_{\mathrm{ps}}^2 - 2 b_{\mathrm{ps}} U_{\mathrm{ps}} + b_{\mathrm{ps}}^2}. \tag{5.6}$$

This model, however, is non-linear in U_{c} and U_{ps}. Therefore a linearization around the equilibrium $U_{\mathrm{c}}^{\mathrm{s}}$ and U_{ps}^{s} where $F_{\mathrm{a}} = 0$ was considered. Applying first-order Taylor expansion to (5.6) yields a linear dynamics of the form

$$\frac{d^2 y}{dt^2} = k_1 y + k_2 u, \tag{5.7}$$

where

$$y = U_{\mathrm{ps}} - U_{\mathrm{ps}}^{\mathrm{s}} \tag{5.8a}$$

$$u = U_{\mathrm{c}} - U_{\mathrm{c}}^{\mathrm{s}} \tag{5.8b}$$

are the deviations of output and input voltages from the selected equilibrium point, respectively. The output deviation of the system y is in the unit of volts and it linearly represents the actual physical distance p. The constants k_1 and k_2 are given by

$$k_1 = 2K_\mathrm{a} \frac{a_\mathrm{ps}^3 {U_\mathrm{c}^\mathrm{s}}^2}{m_\mathrm{b}(U_\mathrm{ps}^\mathrm{s} - b_\mathrm{ps})^3}, \tag{5.9a}$$

$$k_2 = -2K_\mathrm{a} \frac{a_\mathrm{ps}^3 U_\mathrm{c}^\mathrm{s}}{m_\mathrm{b}(U_\mathrm{ps}^s - b_\mathrm{ps})^2}, \tag{5.9b}$$

which, using the values from Table 5.2, amounts to $k_1 = 2250$, $k_2 = 9538$. The system in (5.7) is subsequently converted into a transfer function model via Laplace transform

$$G(s) = \frac{k_2}{s^2 - k_1}. \tag{5.10}$$

The linear dynamics captured in (5.7) can be also transform to the state space model counterpart, specifically

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ k_1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ k_2 \end{bmatrix} u(t), \tag{5.11a}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \tag{5.11b}$$

Based on the transfer function model a suitable stabilizing PID controller was found

$$G_\mathrm{PID}(s) = \frac{13.07s^2 + 1358s + 4.367 \cdot 10^4}{s^2 + 571.4s}. \tag{5.12}$$

The controller in (5.12) is then converted in the a discrete time state space model using $T_\mathrm{s} = 1\,\mathrm{ms}$. Note, that the PID controller (5.12) is implemented as a PSD controller. Next, the system matrices of the closed-loop model were evaluated to

$$A_{\mathrm{CL}} = \begin{bmatrix} 0.6749 & -0.0355 & -0.0008 & -0.0000 \\ 0.0008 & 1.0000 & -0.0000 & -0.0000 \\ -8.8304 & 49.0588 & 0.9727 & 0.0010 \\ -16470.9632 & 97772.4837 & -51.6329 & 0.9751 \end{bmatrix}, \tag{5.13a}$$

$$B_{\mathrm{CL}} = \begin{bmatrix} 0.0008 \\ 0.0000 \\ 0.0296 \\ 56.2470 \end{bmatrix}, \tag{5.13b}$$

$$C_{\mathrm{CL,u}} = \begin{bmatrix} -2418.9076 & 11781.9582 & -7.8599 & 0.0000 \end{bmatrix}, \tag{5.13c}$$

$$D_{\mathrm{CL,u}} = 7.8599, \tag{5.13d}$$

$$C_{\mathrm{CL,y}} = \begin{bmatrix} -5.1447 & 25.0588 & 1.0000 & 0.0000 \end{bmatrix}, \tag{5.13e}$$

$$D_{\mathrm{CL,y}} = 0. \tag{5.13f}$$

The aggregated state space model of the closed-loop system was obtained according to the Section 4.1.2.2.

5.2.2 Laboratory Device

The actual physical system of magnetic levitation used in this work is the CE152 laboratory model, shown in Fig. 5.2. The plant consists of an electromagnetic coil energized by a power amplifier, a linear position sensor and a ferromagnetic ball. The intensity of magnetic field is the manipulated variable and can be directly influenced by voltage electric signal in the range of 0 to $5\,\mathrm{V}$. A measured variable is the reading of the proximity position sensor in the same 0 to $5\,\mathrm{V}$ range.

This system exhibits several properties that result in interesting control design challenges. One of them is the fast dynamics of the system. The time constant of the open loop is approximately $20\,\mathrm{ms}$, which indicates that reasonable sampling rate should be chosen not less than $0.5\,\mathrm{kHz}$ to get high-quality measurements. In order to demonstrate the time efficiency of proposed MPC-based reference governor strategy, an even higher sampling rate of $1\,\mathrm{kHz}$ has been chosen in this work. Other challenging properties of the system are the natural instability and non-linearity. The instability comes form the spatial arrangement of laboratory model, where the levitating object is pulled upwards to the magnetic coil against the force of the gravity. This is the exact opposite of naturally stable magnetic repulsion arrangement used, e.g., by the *maglev* train. Moreover, the system also exhibits a strong nonlinear behavior. As it is obvious from the force balance

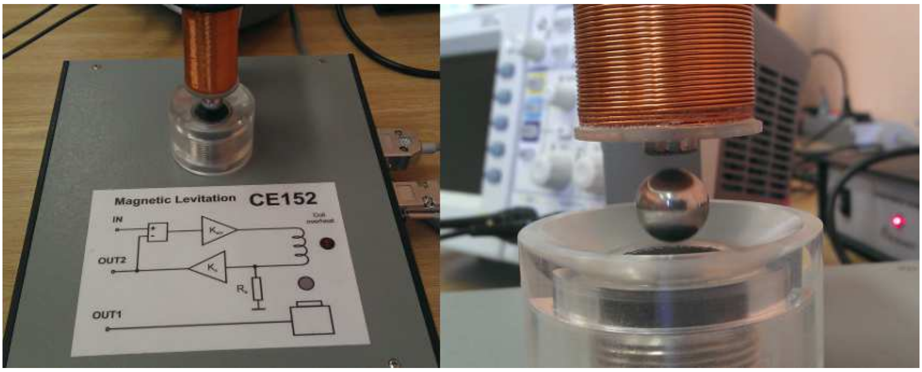


Figure 5.2: Laboratory model of magnetic levitation CE152

equation, the force applied on ball F_a is a sum o gravitational force F_g and force of the magnetic field F_m. While F_g can be considered constant with always present effect, the F_m is a quadratic function of manipulated variable (input voltage U_c) and is present only when nonzero voltage is applied. This results into asymmetric non-linearity, depending on the direction of ball position control.

5.3 Synthesis of the MPC-based Reference Governor

The MPC-based reference governor is structured similarly to (4.20), and implemented as depicted on the Fig. 4.2. The particular MPC strategy is casted as follows

$$\min_{w_0,\ldots,w_{N-1}} \quad \ell_{\mathrm{ss}}(u_k, y_k, w_k) \tag{5.14a}$$

$$\text{s.t.} \quad \widetilde{x}_{k+1} = A_{\mathrm{CL}}\widetilde{x}_k + B_{\mathrm{CL}}w_k, \qquad \forall k \in \mathbb{N}_0^{N-1}, \tag{5.14b}$$

$$u_k = C_{\mathrm{CL,u}}\widetilde{x}_k + D_{\mathrm{CL,u}}w_k, \qquad \forall k \in \mathbb{N}_0^{N-1}, \tag{5.14c}$$

$$y_k = C_{\mathrm{CL,y}}\widetilde{x}_k + D_{\mathrm{CL,y}}w_k, \qquad \forall k \in \mathbb{N}_0^{N-1}, \tag{5.14d}$$

$$u_{\mathrm{min}} \leq u_k \leq u_{\mathrm{max}}, \qquad \forall k \in \mathbb{N}_0^{N-1}, \tag{5.14e}$$

$$y_{\mathrm{min}} \leq y_k \leq y_{\mathrm{max}}, \qquad \forall k \in \mathbb{N}_0^{N-1}, \tag{5.14f}$$

$$w_k = w_{N_\mathrm{c}-1}, \qquad \forall k \in \mathbb{N}_{N_\mathrm{c}}^{N-1}. \tag{5.14g}$$

The objective function (5.14a) is given as

$$\ell_{\mathrm{ss}}(u_k, y_k, w_k) = \sum_{k=0}^{N-1} \Big(\|y_k - r\|^2_{Q_{\mathrm{y}}} + \|\Delta w_k\|^2_{Q_{\mathrm{w}}} + \|\Delta u_k\|^2_{Q_{\mathrm{u}}} + \\ \|w_k - r\|^2_{Q_{\mathrm{wr}}} + \|y_k - w_k\|^2_{Q_{\mathrm{yw}}} \Big). \tag{5.15}$$

The first term of the cost function minimizes the tracking error and forces the plants output, in this case the output voltage, to track a user defined reference r. The second and the third term penalize fluctuations of the optimized setpoints w and the control actions of the primary PSD controller, respectively. The fourth term accounts for the difference between reference r and shaped setpoint w and finally the fifth term penalizes the deviations between predicted output and shaped setpoint for the inner controller. The tuning factors Q_{y}, Q_{w}, Q_{u}, Q_{wr} and Q_{yw} are positive definite matrices of suitable dimensions. The main difference between the structure in (5.14) and e.g. (4.20) is the constraint (5.14g). This constraints represents a move blocking constraint which employs the control horizon $N_{\mathrm{c}} \leq N$ and is used to decrease the number of degrees of freedom and thus make the problem simpler to solve.

he optimization problem in (5.14) is initialized by the vector of parameters

$$\theta = \begin{bmatrix} x_{\mathrm{r}}(t)^{\intercal} & x(t) & w\,(t - T_{\mathrm{s}}) & u\,(t - T_{\mathrm{s}}) & r(t) \end{bmatrix}^{\intercal}, \tag{5.16}$$

where $x_{\mathrm{r}}(t)$ are the states of the inner PSD controller at time t and $x(t)$ are the states of the plant. Moreover, $w\,(t - T_{\mathrm{s}})$ is the value of the optimized reference at the previous sampling instant, required in (5.14a) at $k = 0$ when $w_{-1} = w\,(t - T_{\mathrm{s}})$. Similarly, $u\,(t - T_{\mathrm{s}})$ is the control input generated by the PSD controller at the previous sampling instant that is used as u_{-1} in (5.14a). Because of (5.11), the plant's states $x(t)$ consist of the ball's position $y(t)$ and its speed. Since only the position can be directly measured, the ball's speed is estimated by

$$x^{(2)}(t) = \frac{y(t) - y\,(t - T_{\mathrm{s}})}{T_{\mathrm{s}}}. \tag{5.17}$$

Remark 5.1. The speed, the state no. 2 can be also estimated employing an state observer in a form of a Kalman Filter, but experiments showed, that such a crude estimation as in (5.17) yields satisfactory results. □

Once the MPC-based reference governor is structured, it is solved parametrically as reported in the Section 3.3, hence we obtain a PWA function in the form of

$$W^\star(\theta) = \begin{cases} \alpha_1\theta + \beta_1 & \text{if } \theta \in \mathcal{R}_1 \\ & \vdots \\ \alpha_{n_R}\theta + \beta_{n_R} & \text{if } \theta \in \mathcal{R}_{n_R}, \end{cases} \tag{5.18}$$

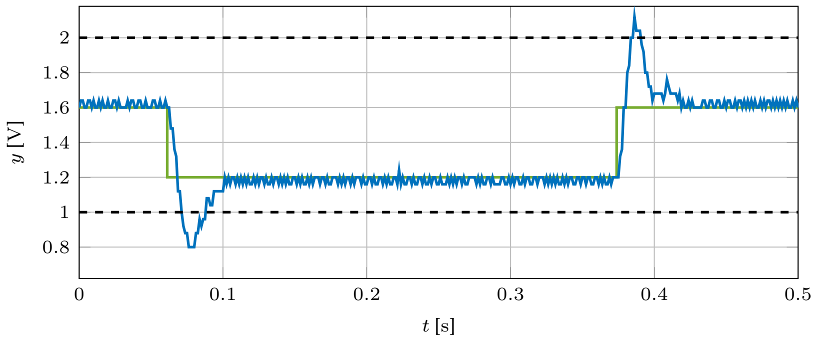
with polyhedral regions $\mathcal{R}$ as in (2.10). The PWA function represeting the MPC-based reference governor was obtained via MPT Toolbox Herceg et al. [2013] in $15\,\mathrm{s}$ on a $2.9\,\mathrm{GHz}$ Core i7 CPU with 12 GB of RAM. The explicit PWA representation of the optimizer $W^\star(\theta)$ in (5.18) consisted of $n_R = 21$ critical regions in a 7 dimensional parametric space, cf. (5.16). The MPT toolbox was subsequently used to synthesize the corresponding binary search tree to speed up the evaluation of the PWA function in (5.18). The total memory footprint of the binary tree was 1392 bytes. This number includes both the separating hyperplanes as well as the parameters of local affine feedback laws α_i, β_i associated to each critical region. The construction of the tree took $5\,\mathrm{s}$. Particular software and hardware implement ion of the MPC-based reference governor on the microprocessor chip can be found in Klaučo et al. [2017].

5.4 Experimental Results

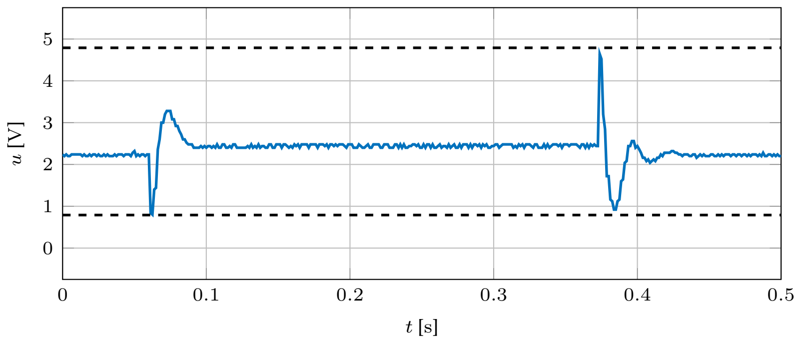
Experimental results demonstrating benefits of the MPC-based reference governor are presented in this section. Two experimental scenarios are considered. In both experiments, the sampling time was chosen as $T_s = 1\,\mathrm{ms}$. Moreover, the reference was periodically switched between $\pm 0.2\,\mathrm{V}$ ($\pm 0.18\,\mathrm{mm}$) around the chosen equilibrium point $U_{ps}^s = 1.4\,\mathrm{V}$ ($p^s = 8.7\,\mathrm{mm}$).

The first experiment concerns controlling the vertical position of the ball solely by the PSD controller (5.12). In this experiment, constraints on the control input were enforced by artificially saturating the control actions. Naturally, the PSD controller provides no a-priori guarantee of satisfying the output constraints. The experimental results are shown in Fig. 5.3. As expected, the control inputs of the PSD controller shown in Fig. 5.3(b) obey the limits on the control action due to the saturation. However, the output constraints are violated, as can be seen in Fig. 5.3(a). Moreover, the control profile exhibits significant under- and overshoots during setpoint changes.

To improve the performance, in the second experiment the MPC-based reference governor was inserted into the loop. The experimental data are visualized in Fig. 5.4. First and foremost, the reference governor shapes the reference in such



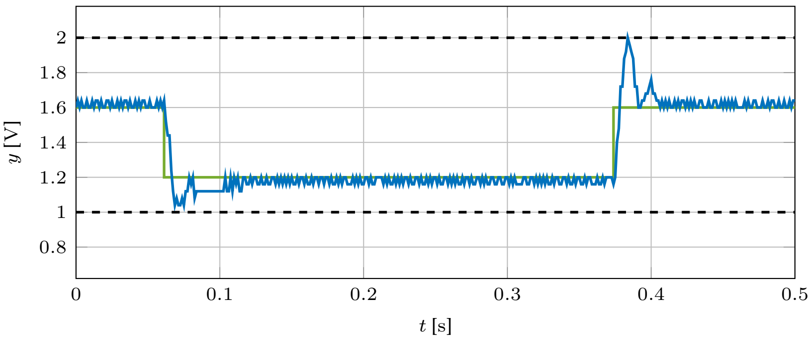
(a) Measurement of the position $y(t)$



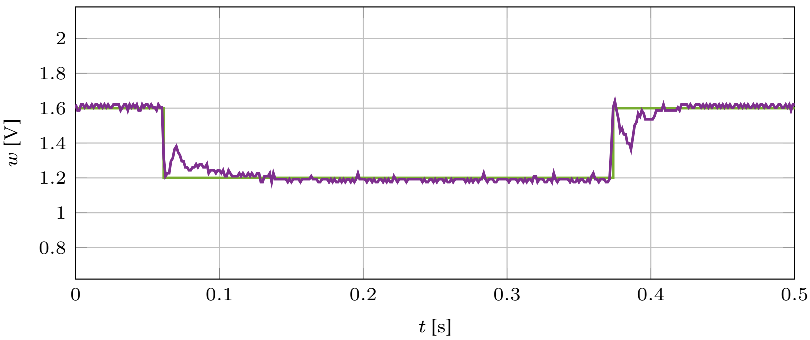
(b) Control action $u(t)$

Figure 5.3: Control by the PSD controller only. The top figure shows the user-defined reference $r(t)$ in green and the ball's position $y(t)$ in blue. The bottom figure shows the control action $u(t)$ of the PSD controller, along with saturation input constraints in dashed-black.

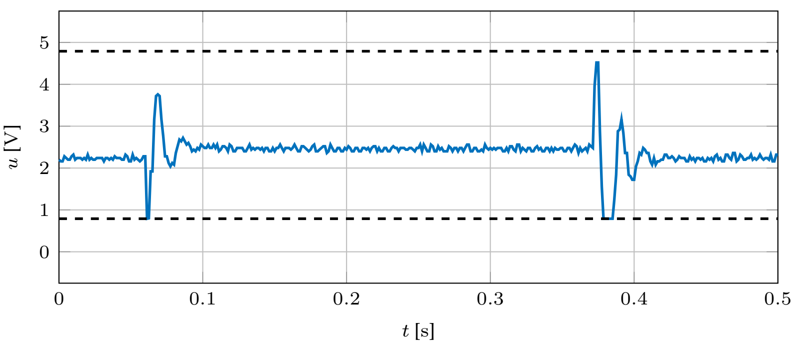
a way that constraints on the controlled output (i.e., on the ball's position) are rigorously enforced, cf. Fig. 5.4(a). This is a consequence of accounting for the constraints directly in the optimization problem, cf. (5.14f). Moreover, the under- and overshoots are considerably reduced. This shows the potential of MPC-based reference governors to significantly improve the control performance. The improvement is due to the optimal modulation of the user-defined reference as shown in Fig. 5.4(b). As can be observed, just small modifications of the reference are required to significantly improve performance. Finally, as can be observed from Fig. 5.4(c), the reference governor shapes the reference in such a way that the inner PSD controller provides satisfaction of input constraints since they are explicitly embedded in (5.14e). The evaluation of the RG feedback law (5.18), encoded as a binary search tree, took 254 µs on average. The best-case



(a) Measurement of the position $y(t)$



(b) Optimally shaped setpoint $w(t)$



(c) Control action $u(t)$ of the PSD controller

Figure 5.4: Control under the reference governor. The top figure shows the measured position of the ball $y(t)$ (blue line), while the reference $r(t)$ is depicted by green. Dashed lines denote output constraints. The middle figure represents the optimally shaped setpoint $w(t)$ (magenta) and the user-defined reference $r(t)$ (green). The bottom figure shows the manipulated variable $u(t)$ and its constraints in dashed-black.

evaluation time was $243\,\mu s$ while the worst case was $266\,\mu s$. These fluctuations are caused by the binary search tree not being perfectly balanced, which is an inherent property of the geometric structure of the parametric solution. As a consequence, different number of steps are required to evaluate the tree for different values of the parameter θ in (5.16).

5.5 Concluding Remarks

In this chapter we show that the performance of the closed-loop system can be enhanced by optimally modulating the setpoint for the inner stabilizing controller. The MPC-based reference governor is based on a state space representation of the closed-loop model, which allows to account for constraints on the control input generated by the inner controller, as well as on the controlled outputs. To implement the MPC strategy on the micro-controller, we choose the explicit MPC realization. The resulting feedback strategy was in a form of a PWA function, which was exported to a binary search tree. By doing so, the MPC strategy was accommodated in approx. 5 kB of memory. Moreover, since the optimal references are precalculated in the form of a PWA function, the on-line implementation effort is modest as well. Specifically, it never exceeded $266\,\mu s$ even on a very simple microprocessor.

The application of explicit MPC was possible only due to the low number of dimension of the parametric space. Next chapter, discusses a system where the number of parametric space increases, due to increased number of states, process variables, and also number of PID loops. The MPC-based governor will be again design based on the state-space representation of the process.

CHAPTER 6

Boiler-Turbine System

Here, the MPC-RG strategy is again based on the state-space modeling of the closed-loop system, as discussed in the Section 4.1.2. This case study involves a more complex model of the closed-loop since it contains three individual PI loops. As a consequence of the size of the MPC-RG, the control problem is solved online, using GUROBI solver. Results published in this chapter are adopted from the paper [Klaučo and Kvasnica, 2017].

6.1 Plant Description and Control Objectives

In this part of the thesis we show how to design a reference governor for a well-known boiler-turbine system, introduced in Åström and Eklund [1972]. It represents a system where fossil fuel is burned to generate steam in a drum boiler. The steam is subsequently fed into the turbine. A schematic representation of the plant is shown in Fig. 6.1. The system features three controlled outputs and three manipulated variables, which have to be operated subject to constraints on their respective amplitudes and their slew rates. Various strategies have been proposed in the literature to control such a plant, ranging from the application of fuzzy MPC Liu and Kong [2013], Li et al. [2012], through data-driven approaches Wu et al. [2014], dynamic matrix control Moon and Lee [2009], up to the application of hybrid MPC techniques Sarailoo et al. [2012], Keshavarz et al. [2010]. Although all aforementioned approaches can substantially improve safety and profitability of the plant operation, they also assume that the existing control architecture (represented by the inner PI/PID controllers) is completely replaced by the new setup. As mentioned in the Introduction, this is not always desired by plant operators.

In this thesis we assume that the individual manipulated variables are controlled by a set of PI loops which, however, do not explicitly take constraints

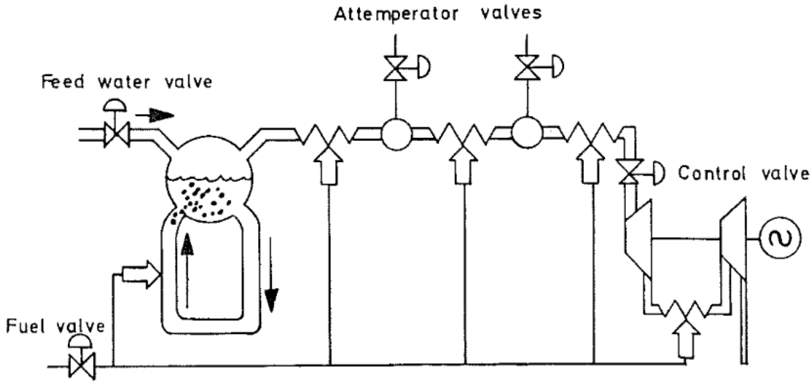


Figure 6.1: The boiler-turbine plant, picture reproduced from Åström and Eklund [1972].

and performance objectives into account. Therefore, an optimization-based reference governor setup is proposed to improve upon these two factors. We show that the inner PI control loops can be modeled as a discrete-time linear time-invariant system. Then, we design a suitable state observer to estimate values of the states which cannot be directly measured, as well as to estimate unmeasured disturbances. Using the model and the estimates we then formulate the optimization problem which predicts the future evolution of the inner closed-loop system and optimizes references provided to the inner PI controllers. Offset-free tracking of output references is furthermore improved by using the concept of disturbance modeling as proposed in the Section 3.2. By means of this case study we illustrate that the proposed concept has two main advantages. First, it allows to keep existing control infrastructure. Second, and more importantly, it enforces a safe and economic operation of the plant. The case study reported in this thesis quantifies the improvement in safety and profitability and also compares it to the scenario where the plant is directly controlled by an MPC controller which bypasses existing PI-based control infrastructure.

The three manipulated variables of the boiler-turbine system are individually controlled by a set of three interconnected PI controllers as suggested by Dimeo and Lee [1995]. Their primary purpose is to stabilize the plant. However, they may exhibit a poor tracking performance due to presence of constraints. Specifically, the amplitude of each control action is constrained in the (normalized) interval $[0,\ 1]$. Additionally, slew rate constraints have to be considered to guarantee a physically safe operation of the plant. Although anti-windup logic can be included into the feedback law to mitigate the influence of min/max constraints on the amplitude, dealing with slew rate constraints requires extensive

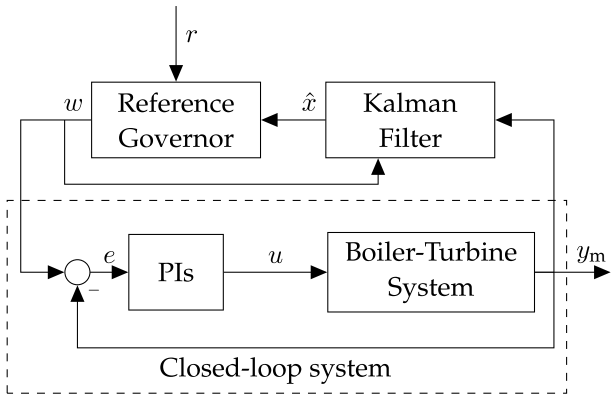


Figure 6.2: Reference governor setup.

controller tuning, which is a tedious procedure without any rigorous guarantees of being successful.

We propose to enforce constraint satisfaction and to improve tracking performance by devising a suitable reference governor. Its purpose is to shape the references provided to individual PI controllers such that even these simple controllers achieve constraints satisfaction and desirable performance. The schematic representation of the proposed strategy is shown in Fig. 6.2. Specifically, the reference governor replaces the user-specified reference r by a shaped reference signal w in such a way that the control actions u generated by the inner PI controllers respect constraints. Moreover, the reference governor improves tracking performance by taking into account predictions of the future evolution of the inner closed-loop system such that the measured plant's output y_m converges to the user-specified reference r without a steady-state offset.

We show that it can be designed as an MPC-like optimization problem solution of which are the optimally shaped references w that enforce constraint satisfaction and optimize tracking performance. Subsequently, in the Section 6.5, the performance of the proposed strategy is compared to a pure PI-based control strategy as well as to full-fledged MPC setup by means of a case study.

6.2 Plant Modeling and Constraints

The mathematical model of the boiler-turbine plant was proposed in Åström and Eklund [1972] as a set of three nonlinear differential equations of the form

$$\frac{\partial p}{\partial t} = -0.0018 u_2 p^{\frac{9}{8}} + 0.9 u_1 - 0.15 u_3, \tag{6.1a}$$

$$\frac{\partial P_{\mathrm{N}}}{\partial t} = (0.073 u_2 - 0.016)\, p^{\frac{9}{8}} - 0.1 P_{\mathrm{N}}, \tag{6.1b}$$

$$\frac{\partial \rho}{\partial t} = \frac{1}{85}\left(141 u_3 - (1.1 u_2 - 0.19)\, p\right). \tag{6.1c}$$

Here, p is the drum pressure in $\mathrm{kg\,cm^{-2}}$, P_{N} denotes the nominal power generated by the turbine in MW, and ρ is the density of the liquid in the boiler in $\mathrm{kg\,m^{-3}}$. These three variables represent the three states of the system. Moreover, the model features three control inputs: u_1 is the fuel flow valve, u_2 represents the steam valve, and u_3 denotes the feed-water valve. The liquid level h in the boiler, represented in m, is given as a nonlinear function of states and inputs in the form

$$h = 6.34 \cdot 10^{-3} p + 4.71 \cdot 10^{-3} \rho + 0.253 u_1 + 0.512 u_2 - 0.014 u_3. \tag{6.2}$$

Only h, p, and P_{N} can be directly measured. The density ρ is an internal state which can only be estimated if it is required by the control strategy.

The three control inputs are subject to min/max bounds on their amplitude with $0 \leq u_i \leq 1$, $i = 1, \ldots, 3$, where 0 represents the fully closed position and 1 stands for the fully open position. In addition, slew rate constraints must be respected as well. They are given by

$$\left|\frac{\partial u_1}{\partial t}\right| \leq 0.007\,\mathrm{s^{-1}}, \tag{6.3a}$$

$$-2\,\mathrm{s^{-1}} \leq \frac{\partial u_2}{\partial t} \leq 0.02\,\mathrm{s^{-1}}, \tag{6.3b}$$

$$\left|\frac{\partial u_3}{\partial t}\right| \leq 0.05\,\mathrm{s^{-1}}. \tag{6.3c}$$

More details about this particular nonlinear model as well as operation of the boiler-turbine unit itself can be found in [Dimeo and Lee, 1995].

The reference governor presented in this part of this thesis is based on a linearization of the nonlinear dynamics in (6.1), followed by conversion of the continuous-time model into the discrete-time domain. Specifically, let

$$x = \begin{bmatrix} p & P_\mathrm{N} & \rho \end{bmatrix}^\intercal - x_\mathrm{L}, \tag{6.4a}$$

$$u = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\intercal - u_\mathrm{L}, \tag{6.4b}$$

$$y = \begin{bmatrix} p & P_\mathrm{N} & h \end{bmatrix}^\intercal - y_\mathrm{L}, \tag{6.4c}$$

denote, respectively, the vector of states, control actions, and measured outputs, expressed as deviations from respective linearization points x_L, u_L, and y_L. Then the linear time-invariant (LTI) approximation of the system in (6.1) in the discrete-time domain is given by

$$x(t + T_\mathrm{s}) = Ax(t) + Bu(t), \tag{6.5a}$$

$$y(t) = Cx(t) + Du(t), \tag{6.5b}$$

where T_s is the sampling time. For this purpose we assume that $T_\mathrm{s} = 2\,\mathrm{s}$.

The linearization point was selected as

$$x_\mathrm{L} = \begin{bmatrix} 107.97 & 66.62 & 428.00 \end{bmatrix}^\intercal, \tag{6.6a}$$

$$u_\mathrm{L} = \begin{bmatrix} 0.34 & 0.69 & 0.44 \end{bmatrix}^\intercal, \tag{6.6b}$$

$$y_\mathrm{L} = \begin{bmatrix} 107.97 & 66.62 & 3.13 \end{bmatrix}^\intercal, \tag{6.6c}$$

and corresponds to the steady state where the turbine generates $66.62\,\mathrm{MW}$ of power with drum pressure of $107.97\,\mathrm{kg\,cm^{-2}}$, and liquid density $428.00\,\mathrm{kg\,m^{-3}}$.

Then, using the first-order Taylor approximation of the nonlinearities in (6.1) and by applying the forward Euler discretization, we arrive at the following matrices of the LTI model in (6.5):

$$A = \begin{bmatrix} 0.9950 & 0 & 0 \\ 0.1255 & 0.8187 & 0 \\ -0.0134 & 0 & 1 \end{bmatrix}, \tag{6.7a}$$

$$B = \begin{bmatrix} 1.7955 & -0.6963 & -0.2993 \\ 0.1168 & 25.6134 & -0.0195 \\ -0.0120 & -2.7733 & 3.3200 \end{bmatrix}, \tag{6.7b}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.0063 & 0 & 0.0047 \end{bmatrix}, \tag{6.7c}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.2530 & 0.5120 & -0.0140 \end{bmatrix}. \tag{6.7d}$$

Worth noting is that the D matrix is non-zero, which is a consequence of direct feedthrough of control inputs in the output equation (6.2).

6.3 Modeling of the Closed-Loop System

We assume that the boiler-turbine system is controlled by three interconnected PI controllers as shown in Fig. 6.3. Their coefficients, as reported by Dimeo and Lee [1995], are as follows:

$$R_1 = \frac{11.119s + 0.003}{s}, \tag{6.8a}$$
$$R_2 = \frac{0.004s + 0.009}{s}, \tag{6.8b}$$
$$R_3 = \frac{1.163s + 0.019}{s}. \tag{6.8c}$$

Furthermore, local gains $k_{12} = 0.0292$, $k_{13} = 0.1344$, $k_{21} = 0.0468$, $k_{23} = 0.0875$, $k_{31} = 0.0842$ and $k_{32} = 0.0699$ are introduced to improve the control performance.

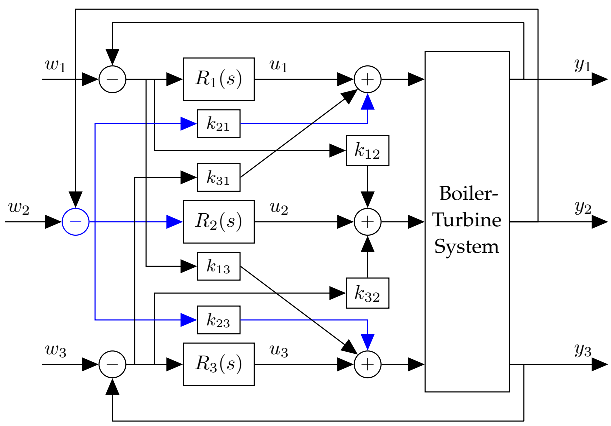


Figure 6.3: Interconnected PI controllers as proposed by Dimeo and Lee [1995].

To deal with the technological constraints (6.3), Dimeo and Lee [1995] have proposed to include additional rate limiters and antiwind-up in the closed-loop logic. Here, however these constraints will be enforced by the reference governor, hence the modeling of the closed-loop system will be done exactly in the same fashion as reported in the Section 4.1.2.1. If we consider the model of the set of PI controllers as in (4.11) and the sampling time $T_s = 2\,\mathrm{s}$, the state space model of the PI controllers is

$$A_{\mathrm{r}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{6.9a}$$

$$B_{\mathrm{r}} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \tag{6.9b}$$

$$C_{\mathrm{r}} = \begin{bmatrix} 0.0033 & 0 & 0 \\ 0 & 0.0093 & 0 \\ 0 & 0 & 0.0186 \end{bmatrix}, \tag{6.9c}$$

$$D_{\mathrm{r}} = \begin{bmatrix} 11.1185 & 0.0468 & 0.0842 \\ 0.0292 & 0.0040 & 0.0699 \\ 0.1344 & 0.0875 & 1.1631 \end{bmatrix}. \tag{6.9d}$$

Note, that the D_{r} is a full matrix, which is a consequence of using local gains k_{ij} in the setup of Fig. 6.3. Next, the matrices of the closed-loop system A_{CL}, B_{CL}, C_{CL}, and D_{CL} are be evaluated in accordance with (4.18). These matrices are then used in the next section for the control strategy design.

6.4 Control Strategies

In this section we will focus on the particular controller design. First, we will design the MPC-RG strategy, where we utilize theoretical basis from the Section 4.1. This section is followed by design of a Direct-MPC strategy, in order to compare the difference in control performance.

6.4.1 MPC-based Reference Governor

The MPC-based reference governor is design based on the linearized model of boiler-turbine unit and its closed-loop representation. Moreover, in order to reject the offset and design model and process mismatch, we introduce the disturbance modeling (Section 3.2) to enforce the reference tracking properties. To further introduce the reader to the implementation details of the time-varying Kalman filter and the MPC-based reference governor, a complex block diagram si drawn in the Fig. 6.4.

The particular design model used for the MPC design as well as the Kalman

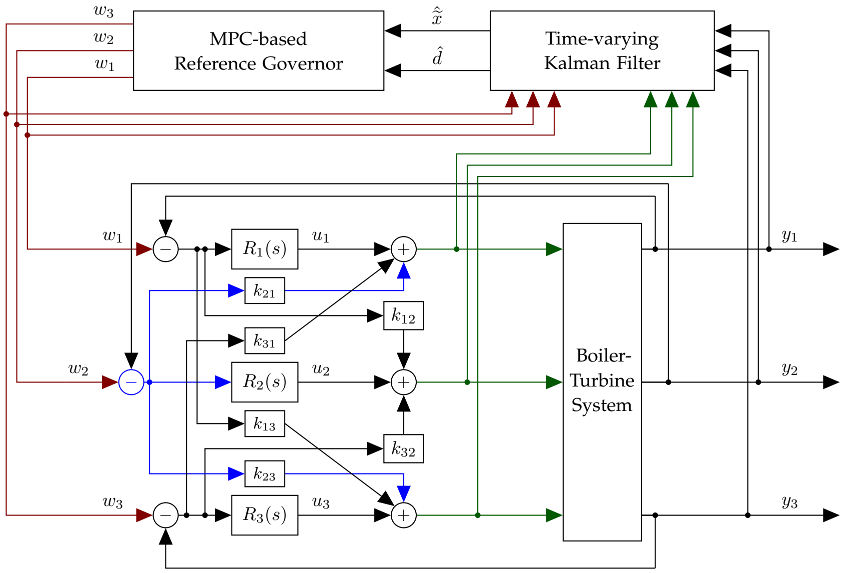


Figure 6.4: MPC-based reference governor implementation with the time-varying Kalman filter.

Filter design is of the form

$$\tilde{x}_{k+1} = A_{\mathrm{CL}}\tilde{x}_k + B_{\mathrm{CL}}w_k, \tag{6.10a}$$

$$u_k = C_{\mathrm{CL,u}}\tilde{x}_k + D_{\mathrm{CL,u}}w_k, \tag{6.10b}$$

$$y_k = C_{\mathrm{CL,y}}\tilde{x}_k + D_{\mathrm{CL,y}}w_k + E_{\mathrm{y}}d_k, \tag{6.10c}$$

$$d_{k+1} = d_k. \tag{6.10d}$$

Next, in order to design the time-varying Kalman filter (Eq. (3.20) and (3.19)) an aggregated disturbance model is derived. Particularly, define a state vector

$$\hat{x}_{\mathrm{e}} = \begin{bmatrix} \hat{\tilde{x}} \\ \hat{d} \end{bmatrix}, \tag{6.11}$$

based on which the model in (6.10) is re-arranged to

$$A_{\mathrm{e}} = \begin{bmatrix} A_{\mathrm{CL}} & 0 \\ 0 & I \end{bmatrix}, \qquad B_{\mathrm{e}} = \begin{bmatrix} B_{\mathrm{CL}} \\ 0 \end{bmatrix}, \tag{6.12a}$$

$$C_{\mathrm{e}} = \begin{bmatrix} C_{\mathrm{CL,u}} & 0 \\ C_{\mathrm{CL,y}} & F \end{bmatrix}, \quad D_{\mathrm{e}} = \begin{bmatrix} D_{\mathrm{CL,u}} \\ D_{\mathrm{CL,y}} \end{bmatrix}. \tag{6.12b}$$

The MPC-based reference governor is designed as a prediction problem over a finite horizon N and it is given by

$$\min_{w_0,\ldots,w_{N-1}} \sum_{k=0}^{N-1} \left(\|y_k - r\|_{Q_\mathrm{y}}^2 + \|\Delta w_k\|_{Q_\mathrm{w}}^2 + \|\Delta u_k\|_{Q_\mathrm{u}}^2 \right) \tag{6.13a}$$

$$\text{s.t. } \widetilde{x}_{k+1} = A_{\mathrm{CL}}\widetilde{x}_k + B_{\mathrm{CL}} w_k, \tag{6.13b}$$

$$u_k = C_{\mathrm{CL,u}}\widetilde{x}_k + D_{\mathrm{CL,u}} w_k, \tag{6.13c}$$

$$y_k = C_{\mathrm{CL,y}}\widetilde{x}_k + D_{\mathrm{CL,y}} w_k + E_\mathrm{y} d_0, \tag{6.13d}$$

$$y_{\min} \leq y_k \leq y_{\max}, \tag{6.13e}$$

$$u_{\min} \leq u_k \leq u_{\max}, \tag{6.13f}$$

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max}, \tag{6.13g}$$

$$\widetilde{x}_0 = \hat{\widetilde{x}}(t), u_{-1} = u(t - T_\mathrm{s}), w_{-1} = w(t - T_\mathrm{s}), d_0 = \hat{d}(t). \tag{6.13h}$$

The optimization problem is initialized by the estimate of the state vector $\hat{\widetilde{x}}(t)$, and by estimate of the disturbance $\hat{d}(t)$, which can be extracted from the Kalman filter procedure via (3.22). Moreover, in the objective function (6.13a) we not only penalize the tracing error by the first term, but also the fluctuations in the shaped reference Δw and the slew rates Δu. Based on that, we require in the initialization variables w_{-1}, and u_{-1}.

6.4.2 Direct-MPC Strategy

The Direct-MPC controller is design in order to compare the MPC-RG strategy with the situation when the PI loops are substituted by the optimal controller. We assume, that the manipulated variables are obtained by solving following optimization problem at each sampling instant

$$\min_{u_0,\ldots,u_{N-1}} \sum_{k=0}^{N-1} \left(\|y_k - r\|_{W_\mathrm{y}}^2 + \|\Delta u_k\|_{W_\mathrm{u}}^2 \right) \tag{6.14a}$$

$$\text{s.t. } x_{k+1} = Ax_k + Bu_k, \tag{6.14b}$$

$$y_k = Cx_k + Du_k + E_\mathrm{y} d_0, \tag{6.14c}$$

$$y_{\min} \leq y_k \leq y_{\max}, \tag{6.14d}$$

$$u_{\min} \leq u_k \leq u_{\max}, \tag{6.14e}$$

$$\Delta u_{\min} \leq \Delta u_k \leq \Delta u_{\max}. \tag{6.14f}$$

Problem (6.14) is similar to (6.13), but we directly optimize the control inputs u of (6.5) instead of the shaped reference w. Moreover, the predictions are based on the model of the nominal system in (6.5). The MPC problem (6.14) can be implemented in a receding horizon fashion similarly as was described in Section 3.1.2. Its initial conditions, i.e., x_0 and d_0, are obtained by estimation by introduction the time-varying Kalman Filter per Section 3.2.

6.5 Case Study

In this section we demonstrate the performance and viability of the proposed reference governor setup in a simulation study involving the nonlinear model of the boiler-turbine system in (6.1). Three cases are considered. In the first case the boiler is controlled solely by the interconnected PI controllers as shown in Fig. 6.3. In the second case the references for the inner PI controllers are shaped in an optimal fashion using the reference governor described in Section 6.4.1 and in Fig. 6.4. We refer to this scenario as the MPC-RG setup. Finally, the last case considers that the plant is directly controlled by an MPC regulator, bypassing the inner PI loops. This scenario will be referred to as Direct-MPC and is discussed in Section 6.4.2, and shown in Fig. 3.1.

In all three scenarios, the sampling time was set to $T_\mathrm{s} = 2\,\mathrm{s}$ and problems (6.13) and (6.14) were formulated for the prediction horizon $N = 30$. The following tuning of penalty matrices used in (6.13a) was used:

$$Q_\mathrm{y} = \mathrm{diag}\left([10^1\ 10^{-2}\ 10^1]\right), \tag{6.15a}$$
$$Q_\mathrm{w} = I_3 \cdot 10^{-4}, \tag{6.15b}$$
$$Q_\mathrm{u} = I_3 \cdot 10^{-3}. \tag{6.15c}$$

The associated Kalman filter was tuned with

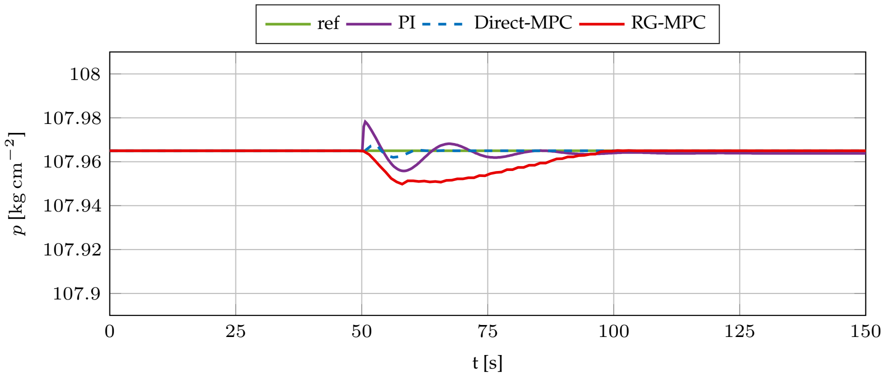
$$P_0 = I_9, \tag{6.16a}$$
$$Q_\mathrm{e} = I_9 \cdot 10^{-4}, \tag{6.16b}$$
$$R_\mathrm{e} = I_6 \cdot 10^{-2}. \tag{6.16c}$$

In case of the Direct-MPC setup, represented by (6.14), we have used

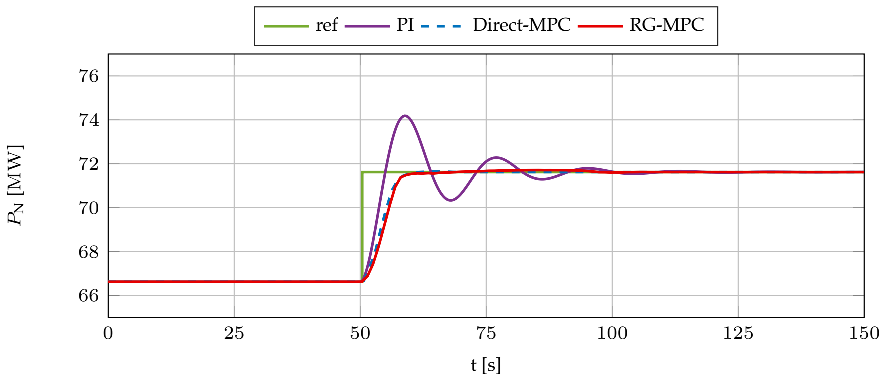
$$W_\mathrm{y} = \mathrm{diag}\left([10^2\ 10^{-1}\ 10^2]\right), \tag{6.17a}$$
$$W_\mathrm{u} = I_3 \cdot 10^{-3}. \tag{6.17b}$$

The tuning of the associated time-varying Kalman filter was chosen as

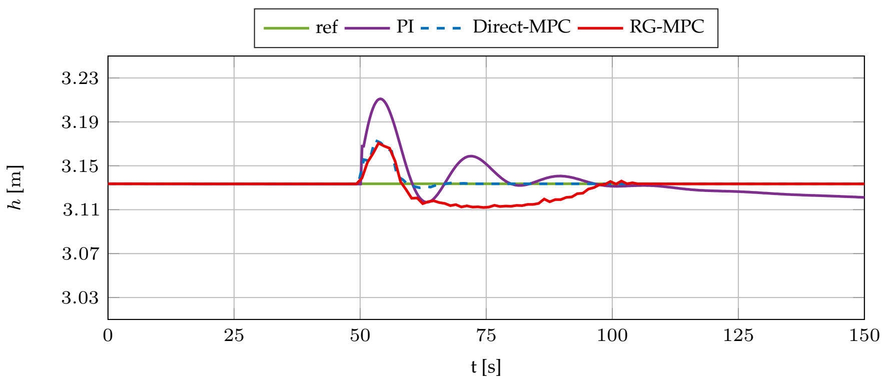
$$P_0 = I_6, \tag{6.18a}$$
$$Q_\mathrm{e} = \mathrm{diag}\left([10^1\ 10^2\ 10^1\ 10^{-1}\ 10^1\ 10^{-1}]\right), \tag{6.18b}$$
$$R_\mathrm{e} = \mathrm{diag}\left([10^1\ 10^1\ 10^{-5}]\right). \tag{6.18c}$$



(a) Drum pressure

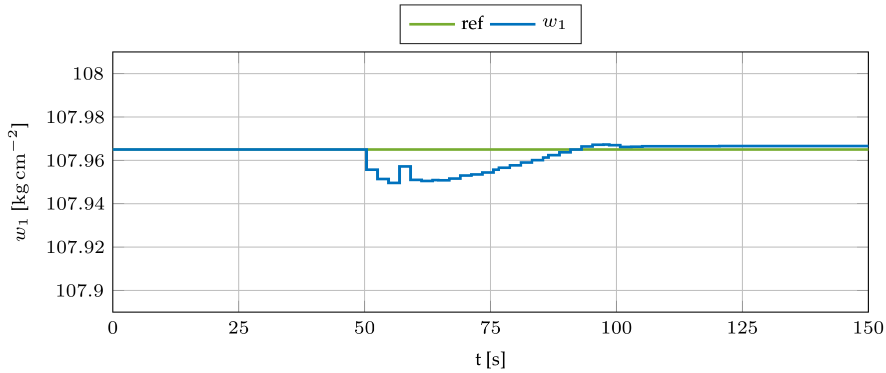


(b) Power generated by turbine

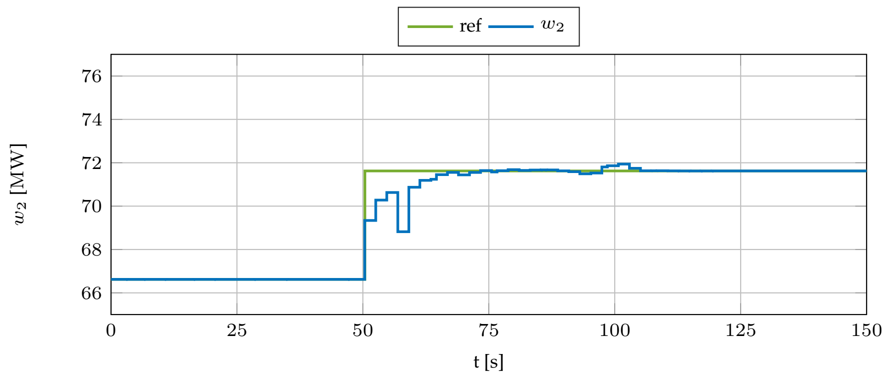


(c) Liquid level in boiler

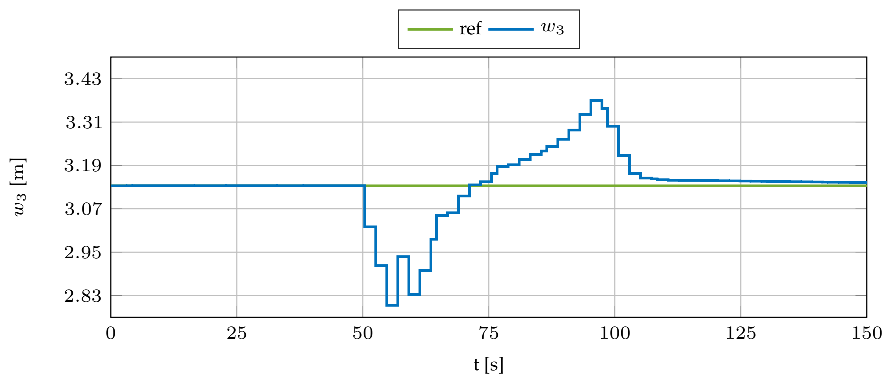
Figure 6.5: Measurements profiles. The green color depicts the reference signal. The pink color show the performance of set of PI controllers, the red lines depicts the performance using RG-MPC strategy, and the blue dashed is used for Direct-MPC setup



(a) Shaped reference for the drum pressure



(b) Shaped reference for the power setpoint



(c) Shaped reference for the liquid level in boiler

Figure 6.6: Shaped references. The green color depicts the reference signal. The blue color show the shaped reference.

Both optimization-based strategies were required to enforce following constraints:

$$u_{\min} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^\intercal - u_{\mathrm{L}}, \tag{6.19a}$$
$$u_{\max} = \begin{bmatrix}1 & 1 & 1\end{bmatrix}^\intercal - u_{\mathrm{L}}, \tag{6.19b}$$

along with constraints on the slew rate of control inputs:

$$\Delta u_{\min} = \begin{bmatrix}-0.014 & -4 & -0.1\end{bmatrix}^\intercal \mathrm{s}^{-1}, \tag{6.20a}$$
$$\Delta u_{\max} = \begin{bmatrix}0.014 & 0.04 & 0.1\end{bmatrix}^\intercal \mathrm{s}^{-1}. \tag{6.20b}$$

In all simulations involving one of the MPC strategies, the respective QP optimization problems were formulated using YALMIP Löfberg [2004] and solved using the Gurobi solver.

The first simulation scenario involves a step change in the requested generated power. Specifically, a $+5\,\mathrm{MW}$ step change w.r.t. to the steady state $P_{\mathrm{N,L}} = 66.62\,\mathrm{MW}$ is performed at time $50\,\mathrm{s}$. The objective is to keep the other two controlled outputs, i.e., the pressure p and the liquid level in the drum h at their steady-state values.

Closed-loop simulation profiles of the controlled outputs of the boiler-turbine system under the three scenarios are shown in Fig. 6.5. As can be observed, the power output signal exhibits noticeable oscillations when the system is controlled purely by the PI controllers. On the other hand, once their respective references are shaped via the reference governor, a much smoother tracking of the power reference is achieved. Worth noting is that by employing the disturbance modeling principle, perfect tracking of the references is achieved despite the controlled system (represented by the nonlinear plant in (6.1)) being different from the prediction model (represented by the LTI system (6.5)).

The corresponding profiles of the shaped references, generated by solving the QP (6.13) at each sampling instant, are shown in Fig. 6.6. Moreover, the performance of the proposed reference governor setup is almost identical to that one of the direct MPC. A similar conclusion can be drawn from the profiles of the liquid level in the boiler. The control actions of all three control strategies are shown in Fig. 6.7. As can be seen, the PI controllers are less aggressive such that they avoid hitting the constraints. The RG-MPC and Direct-MPC strategies, on the other hand, are explicitly aware of the constraints.

To quantify the performance of the three discussed control strategies, we have

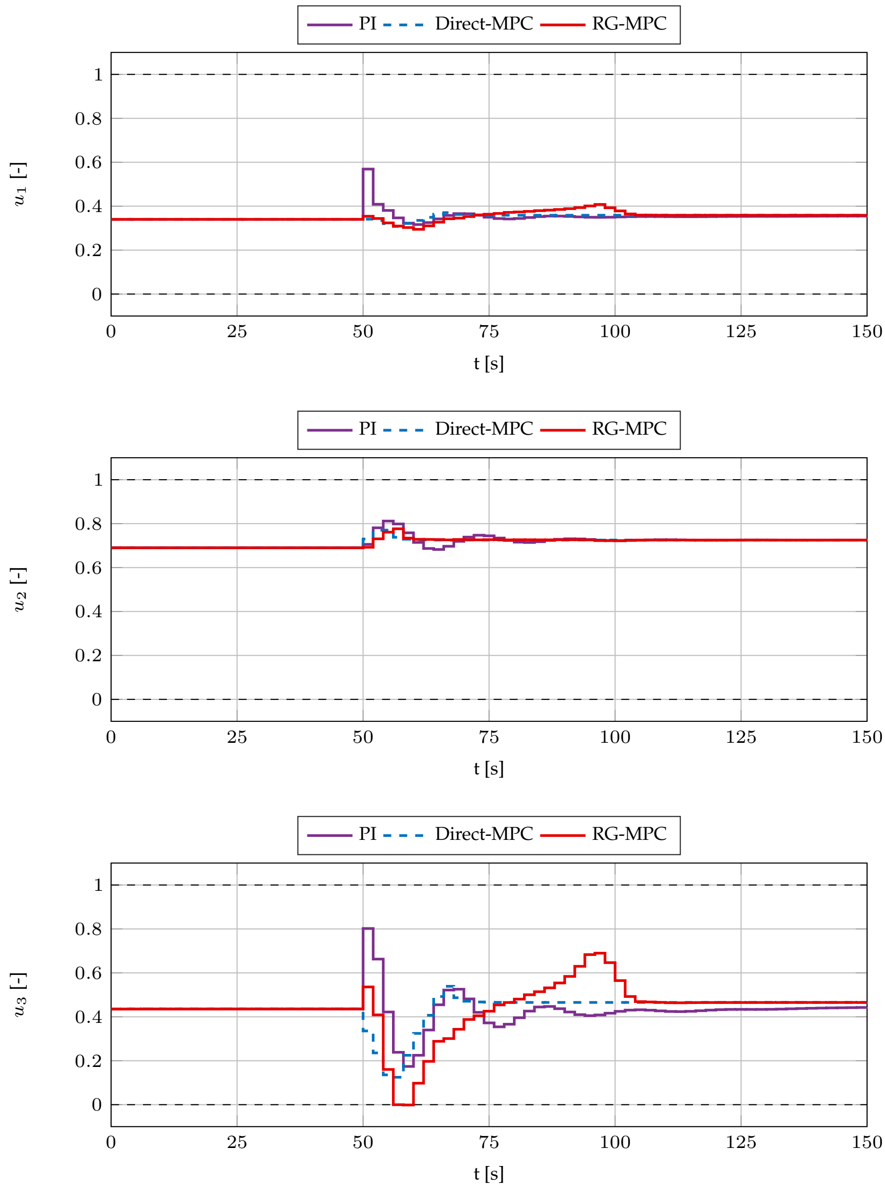


Figure 6.7: Control actions profiles.

evaluated two criteria. The first one, represented by

$$J_{\mathrm{u}} = \frac{1}{N_{\mathrm{sim}}} \sum_{k=1}^{N_{\mathrm{sim}}} \|u_j(k)\|_1, \tag{6.21}$$

quantifies the amount of energy consumed by the individual control strategy. Here, N_{sim} is the number of simulation steps, $u(k)$ denotes the control action at the k-th simulation step, and $\|u(k)\|_1 = \sum_{j=1}^{3} |u_j(k)|$ is the 1-norm of $u(k)$. Similarly,

$$J_{\mathrm{y}} = \frac{1}{\frac{1}{N_{\mathrm{sim}}} \sum_{k=1}^{N_{\mathrm{sim}}} \|y(k) - r(k)\|_1} \tag{6.22}$$

quantifies the tracking performance.

Values of these performance criteria for each of the three considered control strategies are compared graphically in Fig. 6.8. Note that the values are normalized such that the PI strategy has $J_{\mathrm{u}} = 1$ and $J_{\mathrm{y}} = 1$.

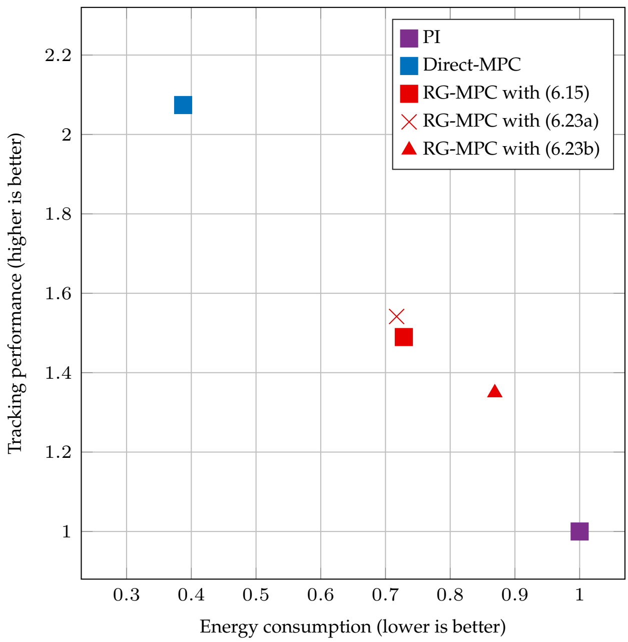


Figure 6.8: Control strategies comparison.

As expected, the pure PI-based strategy exhibits the worst performance both with respect to energy consumption as well as in terms of tracking performance. This is a consequence of the conservative tuning of such PI controllers with the objective to avoid hitting constraints. The best performance is achieved by the Direct-MPC setup, since it bypasses the internal dynamics of the PI controllers. The proposed reference governor-based scheme allows the PI controller to stay as a part of the closed-loop system, but significantly improves their performance by a suitable choice of the shaped references. Specifically, compared to the pure PI strategy, RG-MPC reduces the energy consumption by 27% and improves the tracking performance by 48%.

To demonstrate the effect of tuning parameters used in (6.14a), besides the baseline setup of (6.15) we have also considered two different selection of the weighting matrices:

$$Q_{\mathrm{y}} = \mathrm{diag}\left([10^2\ 10^{-1}\ 10^2]\right),\ Q_{\mathrm{w}} = I_3 \cdot 10^{-4},\ Q_{\mathrm{u}} = I_3 \cdot 10^{-5}, \tag{6.23a}$$

$$Q_{\mathrm{y}} = \mathrm{diag}\left([5 \cdot 10^1\ 10^{-1}\ 10^1]\right),\ Q_{\mathrm{w}} = I_3 \cdot 10^{-5},\ Q_{\mathrm{u}} = I_3 \cdot 10^{-5}. \tag{6.23b}$$

The first tuning in (6.23a) puts a higher emphasis on tracking of references since the Q_{y} matrix is ten times as large as in (6.15). Moreover, Q_{w} and Q_{u} were decreased by a factor of 10. The second choice in (6.23b) allows the shaped reference to follow the user-supplied reference less tightly by further decreasing Q_{w}. As a consequence, the MPC-RG has more "freedom" in the choice of the shaped reference. In all cases, prediction horizon $N = 30$ was used as was the case in the baseline scenario.

The aggregated quality criteria (6.21) and (6.22) evaluated for these re-tuned RG-MPC strategies are depicted visually in Fig. 6.8. Moreover, concrete values of the quality criteria are reported in Table 6.1. Besides the aggregated quality criteria J_{u} and J_{y} computed per (6.21) and (6.22), respectively, the table also reports tracking performance of the three controlled outputs. Specifically, $J_{\mathrm{y,p}}$ related to the quality with which the drum pressure reference is tracked. Similarly, $J_{\mathrm{y},P_{\mathrm{N}}}$ is the quality of the tracking for the nominal power and $J_{\mathrm{y,h}}$ relates to the quality of tracking of the liquid level reference.

Two main conclusions can be drawn from the results in Table 6.1. First, the best overal performance is achieved by using the tuning in (6.23a) which, however, is only marginally better than the baseline tuning of (6.15). The third alternative per (6.23b) is slightly worse with respect to output tracking and significantly worse w.r.t. energy consumption. The second conclusion is that all RG-MPC tunings provide a very good tracking quality of the nominal power P_{N}, even compared to the Direct-MPC setup, cf. the penultimate column of Table 6.1. Specifically, with the tuning in (6.23a), the tracking quality criterion $J_{\mathrm{y},P_{\mathrm{N}}}$ is only by 1.2% worse compared to Direct-MPC. Even with the tuning in (6.23b), which yields the worst tracking of the power reference, the deterioration of

Table 6.1: Quality criteria for various control strategies. Lower values of J_u and higher values of J_y are better. The values are normalized with respect to the PI strategy.

Strategy	J_u	J_y	$J_{\mathrm{y,p}}$	$J_{\mathrm{y},P_\mathrm{N}}$	$J_{\mathrm{y,h}}$
PI	1.0000	1.0000	1.0000	1.0000	1.0000
Direct-MPC	0.3874	2.0741	7.5113	2.0341	5.7237
RG-MPC with (6.15)	0.7284	1.4897	0.7565	1.9189	0.1605
RG-MPC with (6.23a)	0.7173	1.5415	0.7652	2.0095	0.1601
RG-MPC with (6.23b)	0.8689	1.3494	0.7868	1.8572	0.1205

performance is only by 8.7%. On the other hand, by using only the PI controllers, the power tracking performance drops by 50.8% compared to Direct-MPC. Since the nominal power is considered the most important quality in practice, this demonstrates that RG-MPC can indeed significantly improve control quality even when inner PI controllers are included in the loop and can even achieve comparable results to Direct-MPC setups.

6.6 Concluding Remarks

This part of the thesis we have shown how to improve safety and performance of conventional control strategies by a suitable modification of their references. This was achieved by predicting the future evolution of the closed-loop system composed of the controlled plant and a set of interconnected PI controllers. By optimizing over the future predictions we have obtained shaped references which, when fed back to the inner controllers, lead to constraints satisfaction and improved tracking performance. The shaped references were computed by solving a convex quadratic programming problem at each sampling instant. The unmeasured states, as well as disturbances capturing the plant-model mismatch, were estimated using a time-varying Kalman filter. The case study has demonstrated that process safety and performance can indeed be significantly improved. The results of the proposed reference governor setup were also compared to a scenario where the inner controllers are bypassed and the plant is directly controlled by a model predictive control strategy.

By applying optimization on top of existing PI controllers, reference governors can be viewed at as "trojan horses" for application of MPC-based strategies to plants where complete revamp of the control architecture is not desired. This reduces the cost of implementation and, most importantly, allows human operators to stay comfortable with existing control architecture.

As a final note, we recognize two main area, where the MPC-RG can be further improved. The first one includes incorporation of a more detailed prediction model, preferably a nonlinear one. This, on the one hand, would result in further increase of control quality. On the other hand, however, computational obstacles associated with solving nonlinear optimization problems would need to be addressed. The second direction is to assume more complex inner controllers. For instance, one can assume that the coefficients of the PI controllers change based on a look-up table. This would result in a switching in the controller tuning. Such a behavior can be efficiently tackled in the context of hybrid systems [Bemporad and Morari, 1999] at the expense of arriving at a more complex optimization problem.

The chapter 5 and this chapter discussed the design and application of MPC-based reference governor based on the state-space representation of the process and PID controllers. The subsequent chapter moves from the inner PID loop to a loop consisting of "on/off" controller.

CHAPTER 7

Temperature Control in Buildings

This chapter discusses the application of the MPC-based reference governor, which improves the behavior of the closed-loop system consisting of a relay controller. The theoretical basis for this strategy is covered in the Section 4.2. In this chapter, we describe the particular mixed-integer linear problem formulation for the MPC-RG problem (4.47). We test the viability of the strategy on the simulation-based case study involving a thermostatically controlled temperature in the building.

7.1 Challenges in Thermal Comfort Control

The main reason to study the control approaches in connection with the thermal comfort is to decrease the astronomical energy requirements which goes to the heating, ventilation and air-conditioning (HVAC) systems. It has been reported, that almost 40% of the global energy use goes to HVAC systems [Parry et al., 2007]. Therefore it is of imminent importance to design such control systems which attain comfortable thermal conditions while minimizing the energy consumption. Several control strategies which aim at achieving such a goal have been proposed in the literature. One of the most promising approach is represented by Model Predictive Control (MPC), which allows to explicitly account for minimization of the energy consumption while maintaining thermal comfort criteria as presented in [Oldewurtel et al., 2012, Ma et al., 2012], and in [Drgoňa, Kvasnica, Klaučo, and Fikar, 2013]. Suitability of the MPC-based strategies has been well-documented in several scientific publications [Castilla et al., 2011, Cigler et al., 2012b]. In particular, authors in [Široký et al., 2011] report that energy savings under MPC outperform those of intelligent thermostats, ranging from 17% in non-insulated buildings up to 28% in insulated ones.

When we speak about the thermal comfort, we usually mean the temperature inside an office. Almost all strategies aim to achieve suitable temperature conditions while reducing the energy consumption of the heater or the AC unit. However, keeping the temperature constant does not imply that occupants of the building perceive it as the optimal scenario. In fact, there are other factors which influence perception of the thermal comfort including, but not limited to, the relative air humidity, the air movement, the clothing, or the metabolic rate. Therefore the so-called Predicted Mean Vote (PMV) index was introduced by Fanger [1970] as an alternative way of expressing the thermal comfort, standardized in ISO [2006]. The PMV index is a complex, nonlinear relation between various building's parameters and the perceived thermal comfort. The value of the PMV index is dimensionless and its zero value corresponds to most comfortable conditions. Values between ± 1 correspond to slightly cold/warm conditions, PMV $= \pm 2$ expresses cool/warm perception, and values above ± 3 indicate unpleasantly cold/hot conditions in the building. The intriguing property of the PMV index is that it can remain constant even if the indoor temperature changes, for instance when the change of temperature is compensated by the change of air humidity.

Since the PMV index is generally viewed as a superior indicator of thermal comfort, various authors have proposed to formulate MPC with the PMV index in mind [Castilla et al., 2011, Cigler et al., 2012a, Klaučo and Kvasnica, 2014]. Focusing on the PMV index requires also a significant change of the sensor layout, which can be a costly process. One of the aims of this thesis is to provide solutions, which avoid exactly the process of changing the control infrastructure. Therefore, we will look at traditional control setups in residential or office buildings where the temperature measurement is the only variable available to us. In the traditional setups, the entire control scheme consist of a single thermostat connected to central heating system or to an AC unit.

However, conventional thermostats are considered "dumb" since they base their decisions only on the measurements of the indoor temperature. More recently, so-called "intelligent" thermostats started to attract attention due to their ability to provide thermal comfort while mitigating the energy consumption. Examples include, but are not limited to, Nest[1], Emme[2] or Ecobee[3]. Compared to conventional thermostats, these advanced solutions provide higher energy savings by applying algorithms which estimate the habits of household's occupants, especially with respect to occupancy predictions. Although promotional materials of the aforementioned solutions claim a 10% to 15% reduction of the

[1] http://www.nest.com
[2] http://www.getemme.com
[3] http://www.ecobee.com

cost of heating, independent reports are more skeptical. Specifically, Herter [2010] concludes that it is difficult to claim energy savings beyond those obtainable with a conventional programmable thermostat (that has been properly programmed) combined with the provision of energy use information.

Results presented in this thesis originate in Drgoňa, Klaučo, and Kvasnica [2015], in which we aim to improve the behavior of the "dumb" thermostat by an MPC-based reference governor strategy. This strategy will provide an optimal set point to the thermostat, so the thermal comfort will be maintained while we minimize the overall energy expenditures.

7.2 Mathematical Background

The validation of the proposed improvements in control of the indoor temperature involves a simulation study based on a linear time invariant model which employ real historical profiles of disturbances. The prediction model was obtained by zero order hold discretization of continuous time model

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + \tilde{E}d(t), \tag{7.1a}$$

$$T(t) = Cx(t). \tag{7.1b}$$

The system matrices $\tilde{A}$, $\tilde{B}$, $\tilde{E}$ and C were extracted from the ISE toolbox by van Schijndel [2005]:

$$\tilde{A} = 10^{-3} \cdot \begin{bmatrix} -0.020 & 0 & 0 & 0.020 \\ 0 & -0.020 & 0.001 & 0.020 \\ 0 & 0.001 & -0.056 & 0 \\ 1.234 & 2.987 & 0 & -4.548 \end{bmatrix} \tag{7.2a}$$

$$\tilde{B} = 10^{-3} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.003 \end{bmatrix}, \ \tilde{E} = 10^{-3} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.055 & 0 & 0 \\ 0.327 & 0.003 & 0.001 \end{bmatrix}, \tag{7.2b}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{7.2c}$$

The four state variables are as follows: x_1 is the floor temperature, x_2 denotes the internal facade temperature, x_3 is the external facade temperature, and x_4 represents the indoor temperature. All states are in °C. The control input u is the heat flow injected into the building, often denoted as q. According to Fig. 4.5, the heat flow is either zero, i.e. $u_{\text{min}} = 0\,\text{W}$, or equal to $u_{\text{max}} = q_{\text{max}} = 4000\,\text{W}$. The model features three disturbances: d_1 is the external temperature in °C, d_2

is the heat generated by appliances and by the occupants of the building in W, and d_3 is the heat generated by the solar radiation in W. The sampling time of $T_\mathrm{s} = 2.5\,\mathrm{min}$ was used.

7.3 MPC-based Reference Governor Synthesis

The upgrade in the home thermostat system consist of including an MPC-based reference governor. The particular control structure proposed in depicted on the Fig. 7.1. To design the MPC-based reference governor, we adopt the control structure from (4.29). The objective function is set to

$$\ell_\mathrm{z} = \sum_{k=0}^{N-1} u_k, \tag{7.3}$$

which enforces the minimization of the spent energy to achieve comfortable temperatures inside the room. Naturally, if such a objective function would be used, the MPC-based governor will provide such reference w to the thermostat that the injected heat to the room will be $u = 0\,\mathrm{W}$. In the control design for thermal comfort inside residential buildings is standard practice to provide a temperature interval called comfort zone via constraints, i.e, $T \in \{w - \theta, w + \theta\}$. Hence, we will change the constraint (4.29e) to

$$w - \theta \leq T_k \leq w + \theta. \tag{7.4}$$

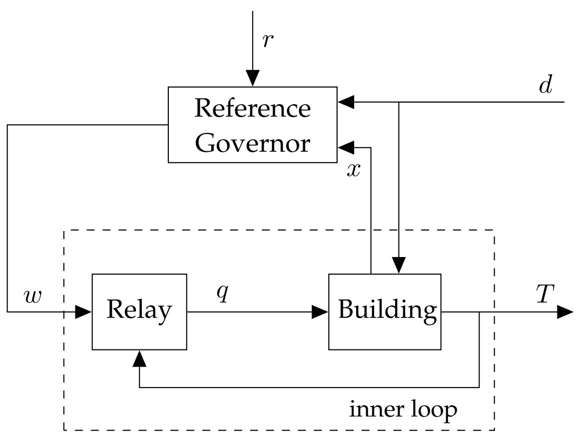


Figure 7.1: Reference governor scheme with a relay-based controller.

Furthermore, we extend the prediction model in (4.29b) by state disturbances as suggested in (7.1). Finally, we modify the thermostat control law given by (4.29f) to

$$u = \begin{cases} q_{\text{max}} & \text{if} \quad z = 1 \\ 0 & \text{if} \quad z = 0. \end{cases} \tag{7.5}$$

Since the variable $u_{\text{min}} = 0$, then the logic rule in (7.5) can be directly translated to

$$u = z q_{\text{max}}, \tag{7.6}$$

hence the part of the MIP (4.47d)-(4.47g) will boil down to (7.6). After these particular modifications, the final structure of the MPC-based reference governor in the form on the optimal control problem is given as

$$
\begin{aligned}
\min_{w_0,\ldots,w_{N-1}} \quad & \sum_{k=0}^{N-1} u_k && \text{(7.7a)} \\
\text{s.t.} \quad & x_{k+1} = A x_k + B u_k + E d_k, && \text{(7.7b)} \\
& T_k = C x_k && \text{(7.7c)} \\
& r - \theta \leq T_k \leq r + \theta, && \text{(7.7d)} \\
& u_k = q_{\text{max}} z_k, && \text{(7.7e)} \\
& w_k + \gamma - T_k \leq M_{\text{a}}(1 - \delta_{\text{a},k}), && \text{(7.7f)} \\
& w_k + \gamma - T_k \geq \epsilon + (m_{\text{a}} - \epsilon)\delta_{\text{a},k}, && \text{(7.7g)} \\
& T_k - w_k + \gamma \leq M_{\text{b}}(1 - \delta_{\text{b},k}), && \text{(7.7h)} \\
& T_k - w_k + \gamma \geq \epsilon + (m_{\text{b}} - \epsilon)\delta_{\text{b},k}, && \text{(7.7i)} \\
& \delta_{1,k} \leq z_k, && \text{(7.7j)} \\
& \delta_{1,k} \leq (1 - \delta_{\text{a},k}), && \text{(7.7k)} \\
& z_k + (1 - \delta_{\text{a},k}) \leq 1 + \delta_{1,k}, && \text{(7.7l)} \\
& \delta_{2,k} \leq (1 - z_k), && \text{(7.7m)} \\
& \delta_{2,k} \leq \delta_{\text{b},k}, && \text{(7.7n)} \\
& (1 - z_k) + \delta_{\text{b},k} \leq 1 + \delta_{2,k}, && \text{(7.7o)} \\
& \delta_{3,k} \geq \delta_{1,k}, && \text{(7.7p)} \\
& \delta_{3,k} \geq \delta_{2,k}, && \text{(7.7q)} \\
& \delta_{3,k} \leq \delta_{1,k} + \delta_{2,k}, && \text{(7.7r)} \\
& z_{k+1} = \delta_{3,k}. && \text{(7.7s)}
\end{aligned}
$$

With the decision variables x_k, T_k, w_k, u_k, all of which are continuous, along with binary variables $\delta_{\text{a},k}$, $\delta_{\text{b},k}$, $\delta_{1,k}$, $\delta_{2,k}$, $\delta_{3,k}$, and z_k. We remind that for the index k holds $k = \{0, \ldots, N-1\}$. Note, that all constraints in (7.7) are linear in the decision variables. Moreover, the objective function (7.7a) is also linear, hence the problem (7.7) can be solved as a mixed-integer linear program. Scalars M_{a}, m_{a}, M_{b}, M_{b} are chosen with respect to the lemma 4.1.

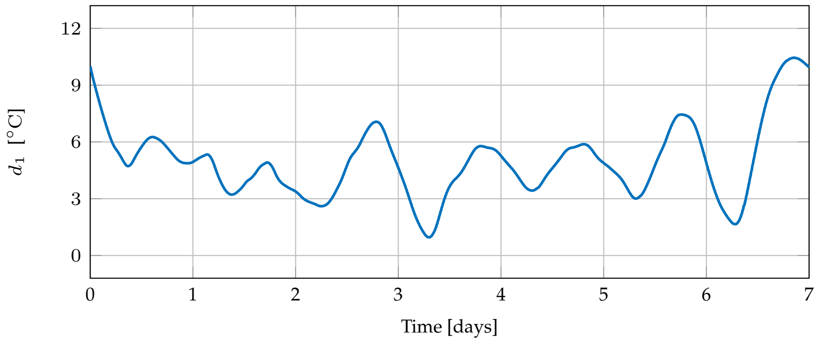
The closed-loop implementation (see Fig. 7.1) of the proposed reference governor strategy is characterized by following steps:

1. Measure (or estimate) current values $x(t)$ of the state variables of the building's thermal model in (7.1) and current disturbances $d(t)$.
2. Create an instance of (7.7) with $x_0 = x(t)$ and $d_0 = d(t)$.
3. Solve (7.7) using a MILP solver and obtain $w_0^\star, \ldots, w_{N-1}^\star$.
4. Communicate $w_0^\star$ to the relay-based thermostat as its setpoint.
5. Repeat at the next sampling instant from step 1.

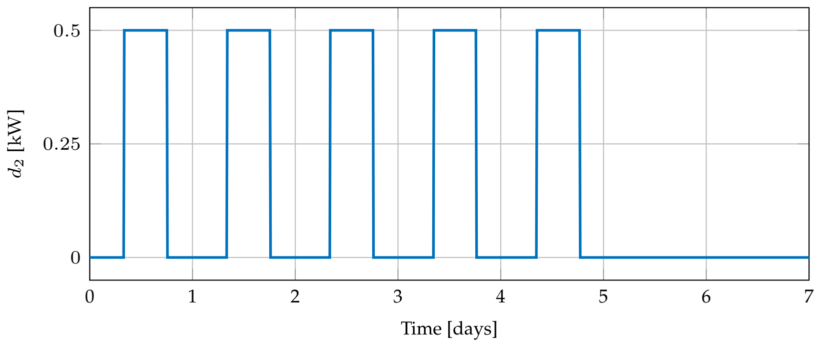
The MILP (7.7) can be formulated in MATLAB using YALMIP [Löfberg, 2004] and solved by GUROBI, CPLEX of MOSEK.

7.4 Performance Comparison

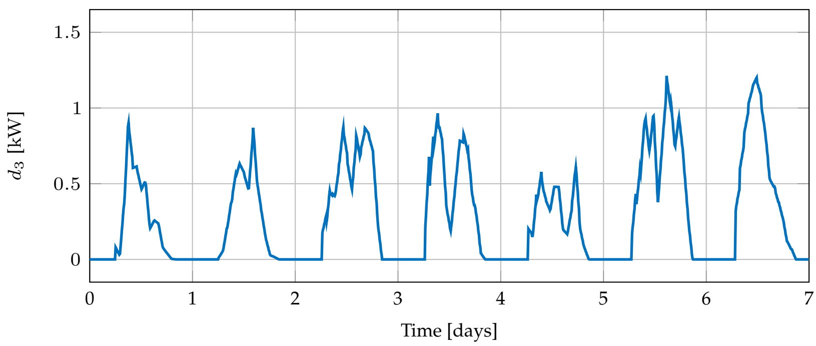
The first simulation study concerns a 1 hour window from 7:00 do 8:00 on the first day for which the disturbance profile can be seen in Fig. 7.2. First, the building was controlled purely by the relay-based thermostat whose setpoint is constantly set to $r = w = 21°\text{C}$. The profile of the indoor temperature can be seen in Fig. 7.3(a). The thermostat switches on and off according to the rules presented in the Section 4.2.1. Throughout the $1\,\text{h}$ window, the thermostat stays in the "on" state for $22.5\,\text{min}$ and consumes $1.5\,\text{kW h}$ of electricity overall, as can be seen in Fig. 7.3(b). Subsequently, the reference governor was enabled and implemented according to Section 7.3. The simulation results are reported in Fig. 7.4. As can be seen, the reference governor modulates the reference in such a way that it allows the thermostat to keep the indoor temperature close to the $r - \theta$ boundary of the thermal comfort zone. As a consequence, less heating energy is required to satisfy the thermal comfort constraint (7.4), cf. Fig. 7.4(c). Specifically, under the optimally modulated setpoint in Fig. 7.4(b), the thermostat stays in the "on" state for $12.5\,\text{min}$ and consumes $0.83\,\text{kW h}$ of energy, hence providing significant reduction of energy consumption compared to the conventional thermostat.



(a) External temperature.

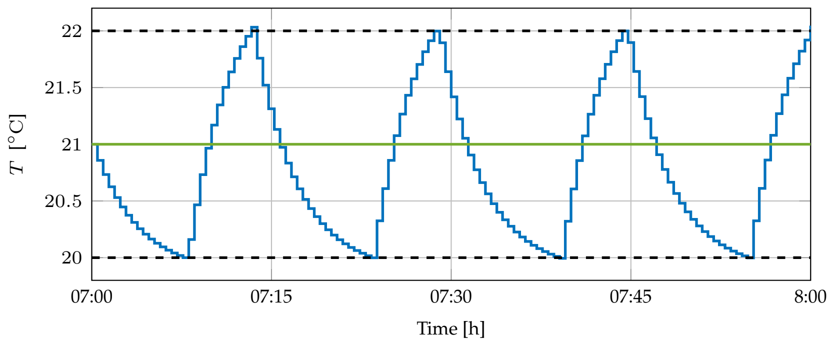


(b) Heat generated by occupancy.

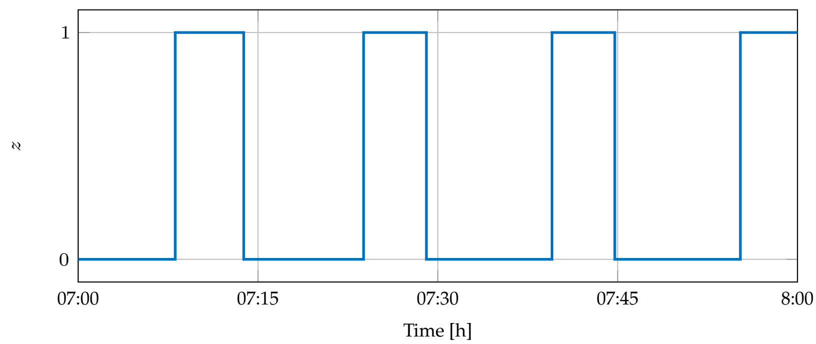


(c) Heat due to solar radiation.

Figure 7.2: 7-days disturbance profiles.

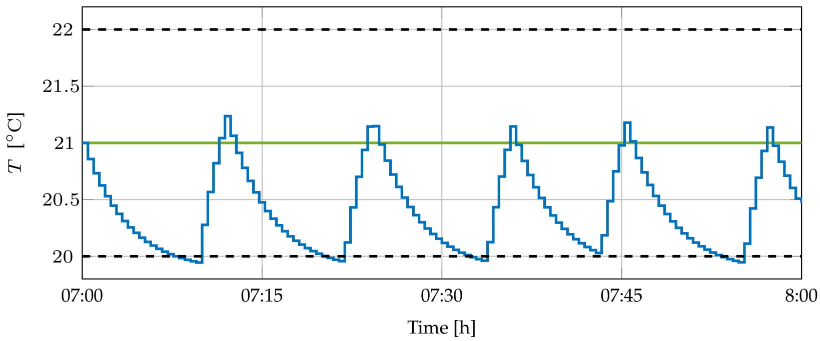


(a) Indoor temperature (blue) and the setpoint (green).

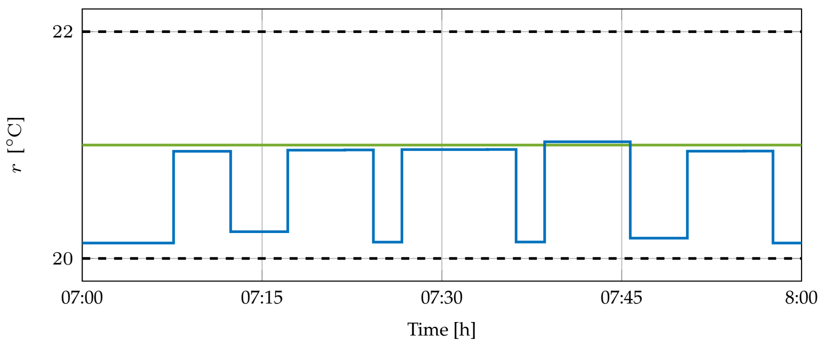


(b) Thermostat on/off state.

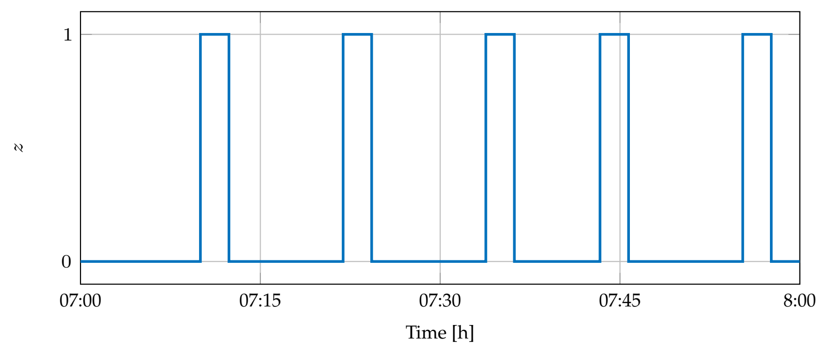
Figure 7.3: Indoor temperature controlled only by the thermostat.



(a) Indoor temperature (blue) and user-specified setpoint (green).



(b) Modulated setpoint r.



(c) Thermostat on/off state.

Figure 7.4: Indoor temperature controlled by a thermostat with the setpoint optimally modulated by the reference governor.

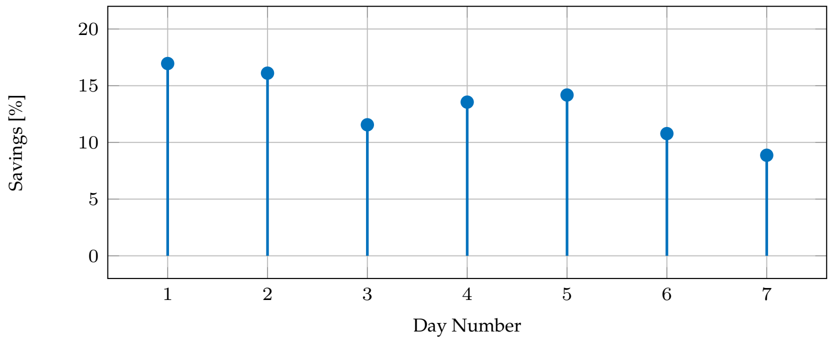


Figure 7.5: Energy savings overview

The second scenario concerns a seven day window with disturbance profiles as in Fig. 7.2. The objective here is to assess performance of the proposed strategy over a longer time span. We have performed closed-loop simulations for two cases. In the first one the building was controlled solely by the thermostat whose setpoint was constantly $w = 21\,°\mathrm{C}$. Then the reference governor was enabled and the simulation was repeated. Performance of both strategies is assessed by measuring the time the heater stays on, i.e., delivering $q = q_{\max}$ power. Daily savings of energy consumption obtained by modulating the reference in an optimal fashion allows to save between 9% to 17% of energy, as reported in Fig. 7.5.

7.5 Concluding Remarks

This chapter presented a design on the MPC-based governor to improve the behavior of conventional thermostats. The supervising controller was designed by taking into account the switching dynamics of the thermostat as well as approximation of the thermal dynamics of the single-zone building. We have shown the transformation of the MPC-like design to the MILP.

CHAPTER 8

Conclusions and Future Remarks

8.1 Thesis Summary

In this thesis, we investigated the properties and benefits of MPC-based reference governors. We have shown, how to derive the models of the closed-loop, upon which is the MPC-based governor designed. In this thesis, we distinguish between three main cases of the closed-loop systems

1. a system with a linear process controlled by set of PID controllers,
2. a system with a linear process controlled by a relay-based controller,
3. a multiple-system composition controlled by decoupled MPC controllers.

For each of the case, we formulate the control problem in the form of a constrained finite time optimization problem. These optimization problems belong to a family of quadratic optimization problems with linear constraints, and in the case of the relay-based inner controller, we arrived at a QP with binary variables.

All of the aforementioned optimization problems can be solved in reasonable time via state-off-the-art solvers. We offer two simulation-based case studies, where we combine the tractable solution of the optimization problems to govern the closed-loop systems. The first case study involves a set of PI controller controlling the boiler-turbine unit, and the second case study shows the improvements in the thermostats behavior. Moreover, we also utilize the parametric programming to obtain the explicit solution to the QP, which was subsequently implemented on a micro-controller to improve the performance of the process of a magnetically suspended ball. Using mentioned case studies and experimental research, we show that the MPC-based governor improves the performance of the closed-loop in regarding constraint enforcement as well as in tracking the reference.

8.2 Future Research Avenues

This last section of this thesis is devoted to establishing future research goals. In here, we will focus on three main fronts. First, is the part of the modeling of the closed-loop systems. By further exploring the types of control loops in the industry, we may find following closed-loop systems, which need to be modeled. They include

- closed-loops with a set of gain-scheduled PID controllers,
- closed-loops with fuzzy logic and other rule-based controllers,
- closed-loops with a combination of several types of controllers.

Depending on the choice of the model of the closed-loop system, the structure of the MPC-based governor changes. With the exploration of models of closed-loop systems is closely related the analysis of the entire MPC-RG strategy with inner controllers. By analysis, we mean rigorous proofs of recursive feasibility and stability of the MPC-RG strategy. This proves to be quite challenging in the case of the on/off inner controller, which leads to an MPC-based governors in the form of a mixed-integer linear programming problem.

The second avenue we may explore is the formulation of optimization problems and its solutions. By adopting procedures from parametric programming, or by reformulating the control problems, we may achieve easier implementability, or we can decrease the demand on computational and memory requirements. To be more specific, lets consider a case of the thermostatically controlled temperature in the building. If the MPC-RG, which is formulated as an MILP, can be accommodated in a simple device like Arduino, owners of residential building and households would be motivated to upgrade their thermostats, without actually modifying the instrumentation. Of course, the saving in terms of one building can be disputable, but the global repercussions can be dramatic. Especially in the explicit MPC-RG strategies arises an opportunity to develop a *tunable* explicit MPC controller. Consider a case that we have a process which is controlled by a PID controller and on top of that we are running MPC-based governor in the form of an explicit MPC controller. If for some reason the tuning of the PID changes, we would need to recompute the solution to the explicit MPC. This can be avoided, by including the constants of the PID controller as parameters of the explicit MPC, which however leads to a bilinear MPC formulation at best.

The third possibility of future research lies in utilizing cloud-based computation services. Here, we suggest moving MPC-based governors to an online service.

Then, the entire communication between the lower layer of control and the process itself would be Internet-dependent. This poses more challenges in Informatics than in the control theory. Moreover, in connection with Internet of Things, this can be a promising research topic. As a final note, the concept of a cloud-based MPC was published by Klaučo et al. [2014], where among other things, an explicit MPC was designed via machine learning procedure. By adopting these machine learning procedures, we may actually provide the MPC and MPC-RG strategies in the cloud as a service to whoever desire to improve control performance of almost any arbitrary process.

Bibliography

K. J. Åström and K. Eklund. A simplified non-linear model of a drum boiler-turbine unit. *International Journal of Control*, 16(1):145–169, 1972.

Karl Johan Åström and Tore Hägglund. *Advanced PID Control*. ISA - The Instrumentation, Systems, and Automation Society; Research Triangle Park, NC 27709, 2006. ISBN 978-1-55617-942-6.

Thomas Bächle, Sebastian Hentzelt, and Knut Graichen. Nonlinear model predictive control of a magnetic levitation system. *Control Engineering Practice*, 21(9):1250 – 1258, 2013. ISSN 0967-0661. doi: http://dx.doi.org/10.1016/j.conengprac.2013.04.009. URL http://www.sciencedirect.com/science/article/pii/S0967066113000828.

A. Bemporad. Reference governor for constrained nonlinear systems. *Automatic Control, IEEE Transactions on*, 43(3):415–419, 1998. ISSN 0018-9286. doi: 10.1109/9.661611.

A. Bemporad. A multiparametric quadratic programming algorithm with polyhedral computations based on nonnegative least squares. *IEEE Transactions on Automatic Control*, 60(11):2892–2903, 2015.

A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.

A. Bemporad and E. Mosca. Constraint fulfilment in feedback control via predictive reference management. In *Control Applications, Proceedings of the Third IEEE Conference on*, pages 1909–1914, 1994.

A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1):3 – 20, 2002. ISSN 0005-1098. doi: http://dx.doi.org/10.1016/S0005-1098(01)00174-1.

D.F. Berdy, D.J. Valentino, and D. Peroulis. Kinetic energy harvesting from human walking and running using a magnetic levitation energy harvester. *Sensors and Actuators A: Physical*, 222:262 – 271, 2015. ISSN 0924-4247. doi: http://dx.doi.org/10.1016/j.sna.2014.12.006.

F. Borrelli. *Constrained Optimal Control of Linear and Hybrid Systems*, volume 290. Springer-Verlag, 2003.

F. Borrelli, P. Falcone, J. Pekar, and G. Stewart. Reference governor for constrained piecewise affine systems. *Journal of Process Control*, 19(8):1229 – 1237, 2009. ISSN 0959-1524.

F. Borrelli, A. Bemporad, and M. Morari. *Predictive control for linear and hybrid systems*. In preparation, 2015. URL http://www.mpc.berkeley.edu/mpc-course-material.

Francesco Borrelli, Mato Baotić, Jaroslav Pekar, and Greg Stewart. On the computation of linear model predictive control laws. *Automatica*, 46(6):1035 – 1041, 2010. ISSN 0005-1098. doi: http://dx.doi.org/10.1016/j.automatica.2010.02.031. URL http://www.sciencedirect.com/science/article/pii/S0005109810001032.

S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, New York, USA, 7th edition, 2009.

E. F. Camacho and C. Bordons. *Model Predictive Control*. Springer, 2nd edition, 2007.

M. Castilla, J.D. Álvarez, M. Berenguel, F. Rodríguez, J.L. Guzmán, and M. Pérez. A comparison of thermal comfort predictive control strategies. *Energy and Buildings*, 43(10):2737 – 2746, 2011. ISSN 0378-7788. doi: http://dx.doi.org/10.1016/j.enbuild.2011.06.030. URL http://www.sciencedirect.com/science/article/pii/S0378778811002799.

J. Cigler, S. Prívara, Z. Váňa, E. Žáčeková, and L. Ferkl. Optimization of predicted mean vote index within model predictive control framework: Computationally tractable solution. *Energy and Buildings*, 52(0):39 – 49, 2012a. ISSN 0378-7788. doi: http://dx.doi.org/10.1016/j.enbuild.2012.05.022. URL http://www.sciencedirect.com/science/article/pii/S0378778812002770.

J. Cigler, S. Prívara, Z. Váňa, E. Žáčeková, and L. Ferkl. Optimization of predicted mean vote index within model predictive control framework: Computationally tractable solution. *Energy and Buildings*, 52(0):39 – 49, 2012b. ISSN 0378-7788. doi: http://dx.doi.org/10.1016/j.enbuild.2012.05.

022. URL http://www.sciencedirect.com/science/article/pii/S0378778812002770.

M. Conforti, G. Cornuejols, and G. Zambelli. *Integer Programming*. Springer Publishing Company, Incorporated, 2014. ISBN 3319110071, 9783319110073.

C. R. Cutler and B. L. Ramaker. Dynamic Matrix Control. In *AIChE 86th National Meeting*, Houston, Texas, 1979.

R. Dimeo and K.Y. Lee. Boiler-turbine control system design using a genetic algorithm. *Energy Conversion, IEEE Transactions on*, 10(4):752–759, 1995. ISSN 0885-8969.

J. Drgoňa, M. Kvasnica, M. Klaučo, and M. Fikar. Explicit stochastic mpc approach to building temperature control. In *IEEE Conference on Decision and Control*, pages 6440–6445, Florence, Italy, 2013.

J. Drgoňa, M. Klaučo, and M. Kvasnica. MPC-Based Reference Governors for Thermostatically Controlled Residential Buildings. In *54rd IEEE Conference on Decision and Control*, volume 54, Osaka, Japan, December 2015. URL http://www.kirp.chtf.stuba.sk/assets/publication_info.php?id_pub=1676.

J. Drgoňa, M. Klaučo, F. Janeček, and M. Kvasnica. Optimal control of a laboratory binary distillation column via regionless explicit mpc. *Computers & Chemical Engineering*, pages 139–148, 2017. doi: 10.1016/j.compchemeng.2016.10.003. URL http://www.kirp.chtf.stuba.sk/assets/publication_info.php?id_pub=1768.

P. Fanger. Thermal comfort. analysis and applications in environmental engineering. *Thermal comfort. Analysis and applications in environmental engineering.*, 1970.

S. Folea, C. I. Muresan, R. De Keyser, and C. M. Ionescu. Theoretical analysis and experimental validation of a simplified fractional order controller for a magnetic levitation system. *IEEE Transactions on Control Systems Technology*, 24(2):756–763, March 2016. ISSN 1063-6536. doi: 10.1109/TCST.2015.2446496.

T. Geyer, F.D. Torrisi, and M. Morari. Optimal complexity reduction of polyhedral piecewise affine systems. *Automatica*, 44(7):1728–1740, July 2008.

E. G. Gilbert and I Kolmanovsky. Fast reference governors for systems with state and control constraints and disturbance inputs. *International Journal of Robust Nonlinear Control*, 1999.

Elmer Gilbert and Ilya Kolmanovsky. Nonlinear tracking control in the presence of state and control constraints: a generalized reference governor. *Automatica*, 38(12):2063 – 2073, 2002. ISSN 0005-1098. doi: 10.1016/S0005-1098(02)00135-8. URL http://www.sciencedirect.com/science/article/pii/S0005109802001358.

T. Glück, W. Kemmetmüller, C. Tump, and A. Kugi. A novel robust position estimator for self-sensing magnetic levitation systems based on least squares identification. *Control Engineering Practice*, 19(2):146 – 157, 2011. ISSN 0967-0661. doi: http://dx.doi.org/10.1016/j.conengprac.2010.11.003. URL http://www.sciencedirect.com/science/article/pii/S0967066110002509.

W. L. Haisler, D. M. Timm, J. A. Gage, H. Tseng, T. C. Killian, and G. R. Souza. Three-dimensional cell culturing by magnetic levitation. *Nature Protocols*, 8: 1940 – 1949, 2013. ISSN 1754-2189. doi: http://dx.doi.org/10.1038/nprot.2013.125.

M. Herceg, M. Kvasnica, C. Jones, and M. Morari. Multi-parametric toolbox 3.0. In *2013 European Control Conference*, pages 502–510, 2013. URL http://www.kirp.chtf.stuba.sk/publication_info.php?id_pub=1428.

K. Herter. Small business demand response with communicating thermostats: Smud's summer solutions research pilot. *Lawrence Berkeley National Laboratory*, 2010.

J. Holaza, B. Takács, M. Kvasnica, and S. Di Cairano. Nearly optimal simple explicit mpc controllers with stability and feasibility guarantees. *Optimal Control Applications and Methods*, 35(6), July 2015. doi: 10.1002/oca.2131.

J. Holaza, M. Klaučo, and M. Kvasnica. Solution techniques for multi-layer mpc-based control strategies (accepted). In *Preprints of the 20th IFAC World Congress, France*, pages –, 2017.

7730 ISO. *EN ISO 7730:2006: Ergonomics of the Thermal Environment - Analytical Determination and Interpretation of Using Calculation of the PMV and PPD Indices and Local Thermal Comfort Criteria*. ISO, Geneva, Switzerland, 2006.

M. Kalúz, M. Klaučo, and M. Kvasnica. Real-time implementation of a reference governor on the arduino microcontroller. In M. Fikar and M. Kvasnica, editors, *Proceedings of the 20th International Conference on Process Control*, pages 350–356, Štrbské Pleso, Slovakia, June 9-12, 2015 2015. Slovak University of Technology in Bratislava, Slovak Chemical Library. doi: 10.1109/PC.2015.7169988. URL http://www.kirp.chtf.stuba.sk/assets/publication_info.php?id_pub=1604.

M. Keshavarz, M. Barkhordari Yazdi, and M.R. Jahed-Motlagh. Piecewise affine modeling and control of a boiler–turbine unit. *Applied Thermal Engineering*, 30(8–9):781 – 791, 2010. ISSN 1359-4311. doi: http://dx.doi.org/10.1016/j.applthermaleng.2009.11.009. URL http://www.sciencedirect.com/science/article/pii/S1359431109003378.

M. Klaučo and M. Kvasnica. Explicit mpc approach to pmv-based thermal comfort control. In *53rd IEEE Conference on Decision and Control*, volume 53, pages 4856–4861, Los Angeles, California, USA, December 15-17, 2014 2014.

M. Klaučo and M. Kvasnica. Control of a boiler-turbine unit using mpc-based reference governors. *Applied Thermal Engineering*, pages 1437–1447, 2017. doi: 10.1016/j.applthermaleng.2016.09.041. URL http://www.kirp.chtf.stuba.sk/assets/publication_info.php?id_pub=1758.

M. Klaučo, J. Drgoňa, M. Kvasnica, and S. Di Cairano. Building temperature control by simple mpc-like feedback laws learned from closed-loop data. In *Preprints of the 19th IFAC World Congress Cape Town (South Africa) August 24 - August 29, 2014*, pages 581–586, 2014.

M. Klaučo, M. Kalúz, and M. Kvasnica. Real-time implementation of an explicit mpc-based reference governor for control of a magnetic levitation system. *Control Engineering Practice*, pages 99–105, 2017. doi: 10.1016/j.conengprac.2017.01.001. URL http://www.kirp.chtf.stuba.sk/assets/publication_info.php?id_pub=1789.

M. Kvasnica. *Real-Time Model Predictive Control via Multi-Parametric Programming: Theory and Tools*. VDM Verlag, Saarbruecken, November 2009. URL http://www.kirp.chtf.stuba.sk/publication_info.php?id_pub=893.

M. Kvasnica and M. Fikar. Clipping-based complexity reduction in explicit mpc. *IEEE Transactions On Automatic Control*, 57(7):1878–1883, July 2012. ISSN 0018-9286.

M. Kvasnica, B. Takács, J. Holaza, and S. Di Cairano. On region-free explicit model predictive control. In *54rd IEEE Conference on Decision and Control*, volume 54, pages 3669–3674, Osaka, Japan, December 15-18, 2015 2015.

Hyung-Woo Lee, Ki-Chan Kim, and Ju Lee. Review of maglev train technologies. *IEEE Transactions on Magnetics*, 42(7):1917–1925, July 2006. ISSN 0018-9464. doi: 10.1109/TMAG.2006.875842.

Y. Li, J. Shen, K. Lee, and X. Liu. Offset-free fuzzy model predictive control of a boiler–turbine system based on genetic algorithm. *Simulation Modelling Practice and Theory*, 26:77 – 95, 2012. ISSN 1569-190X. doi: http://dx.doi.org/

10.1016/j.simpat.2012.04.002. URL http://www.sciencedirect.com/science/article/pii/S1569190X12000524.

C. M. Lin, M. H. Lin, and C. W. Chen. Sopc-based adaptive pid control system design for magnetic levitation system. *IEEE Systems Journal*, 5(2):278–287, June 2011. ISSN 1932-8184. doi: 10.1109/JSYST.2011.2134530.

C. M. Lin, Y. L. Liu, and H. Y. Li. Sopc-based function-link cerebellar model articulation control system design for magnetic ball levitation systems. *IEEE Transactions on Industrial Electronics*, 61(8):4265–4273, Aug 2014. ISSN 0278-0046. doi: 10.1109/TIE.2013.2288201.

X. Liu and X. Kong. Nonlinear fuzzy model predictive iterative learning control for drum-type boiler–turbine system. *Journal of Process Control*, 23(8):1023 – 1040, 2013. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/j.jprocont.2013.06.004. URL http://www.sciencedirect.com/science/article/pii/S0959152413001261.

J. Löfberg. YALMIP : A Toolbox for Modeling and Optimization in MATLAB. In *Proc. of the CACSD Conference*, Taipei, Taiwan, 2004. Available from http://users.isy.liu.se/johanl/yalmip/.

Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves. Model predictive control for the operation of building cooling systems. *Control Systems Technology, IEEE Transactions on*, 20(3):796–803, 2012.

J. M. Maciejowski. *Predictive Control with Constraints*. PEARSON Prentice-Hall, 2002.

D. Q. Mayne. Model predictive control: Recent developments and future promise. *Automatica*, 50(12):2967 – 2986, 2014. ISSN 0005-1098. doi: http://dx.doi.org/10.1016/j.automatica.2014.10.128. URL http://www.sciencedirect.com/science/article/pii/S0005109814005160.

D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789 – 814, 2000. ISSN 0005-1098. doi: 10.1016/S0005-1098(99)00214-9. URL http://www.sciencedirect.com/science/article/pii/S0005109899002149.

U. Moon and K.Y. Lee. Step-Response Model Development for Dynamic Matrix Control of a Drum-Type Boiler-Turbine System. *Energy Conversion, IEEE Transactions on*, 24(2):423–430, June 2009. ISSN 0885-8969. doi: 10.1109/TEC.2009.2015986.

E. Mosca. Nonlinear predictive command governors for constrained tracking. In *Colloquium on Automatic Control*, pages 55–76, 1996.

K. R. Muske. Steady-state target optimization in linear model predictive control. In *American Control Conference, 1997. Proceedings of the 1997*, volume 6, pages 3597–3601 vol.6, Jun 1997. doi: 10.1109/ACC.1997.609493.

K. R. Muske and T. A. Badgwell. Disturbance modeling for offset-free linear model predictive control. *Journal of Process Control*, 12(5):617 – 632, 2002. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/S0959-1524(01)00051-8. URL http://www.sciencedirect.com/science/article/pii/S0959152401000518.

K. R. Muske and J. B. Rawlings. Model predictive control with linear models. *AIChE Journal*, 39(2), February 1993.

J. Nocedal and S. J. Wright. *Numerical Optimization*. Springer, New York, 2nd edition, 2006.

R. Oberdieck, N. Diangelakis, I. Nascu, M. Papathanasiou, M. Sun, S. Avraamidou, and E. Pistikopoulos. On multi-parametric programming and its applications in process systems engineering. *Chemical Engineering Research and Design*, 2016a.

R. Oberdieck, N. A Diangelakis, M. Papathanasiou, I. Nascu, and E. Pistikopoulos. POP–Parametric Optimization Toolbox. *Industrial & Engineering Chemistry Research*, 55(33):8979–8991, 2016b.

F. Oldewurtel, A. Parisio, C.N. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari. Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings*, 45:15–27, 2012.

G. Pannocchia. Robust disturbance modeling for model predictive control with application to multivariable ill-conditioned processes. *Journal of Process Control*, 13(8):693 – 701, 2003. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/S0959-1524(02)00134-8. URL http://www.sciencedirect.com/science/article/pii/S0959152402001348.

G. Pannocchia and J. B. Rawlings. Disturbance models for offset-free model-predictive control. *AIChE Journal*, 49(2), February 2003.

M. Parry, O. Canziani, J. Palutikof, P. van der Linden, and C. Hanson. *Climate change 2007: impacts, adaptation and vulnerability*. Intergovernmental Panel on Climate Change, 2007.

G. Prasath and J.B. Jørgensen. Soft constraints for robust MPC of Uncertain systems. In *International Symposium on Advanced Control of Chemical Processes*, 2009.

S. J. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733 – 764, 2003. ISSN 0967-0661. doi: 10.1016/S0967-0661(02)00186-7. URL http://www.sciencedirect.com/science/article/pii/S0967066102001867.

J. B. Rawlings and D. Q. Mayne. Model predictive control: Theory and design, 2009.

J. Richalet, A. Rault, J.L. Testud, and J. Papon. Model predictive heuristic control. *Automatica*, 14(5):413 – 428, 1978. ISSN 0005-1098. doi: http://dx.doi.org/10.1016/0005-1098(78)90001-8. URL http://www.sciencedirect.com/science/article/pii/0005109878900018.

M. Sarailoo, B. Rezaie, and Z. Rahmani. MLD model of boiler-turbine system based on pwa linearization approach. *International Journal of Control Science and Engineering*, 2:88–92, 2012. doi: 10.5923/j.control.20120204.06.

T. Schuhmann, W. Hofmann, and R. Werner. Improving operational performance of active magnetic bearings using kalman filter and state feedback control. *IEEE Transactions on Industrial Electronics*, 59(2):821–829, Feb 2012. ISSN 0278-0046. doi: 10.1109/TIE.2011.2161056.

L. R. E. Shead, K. R. Muske, and J. A. Rossiter. Adaptive steady-state target optimization using iterative modified gradient-based methods in linear non-square mpc. In *American Control Conference, 2009. ACC '09.*, pages 2558–2563, June 2009. doi: 10.1109/ACC.2009.519829.

L. R. E. Shead, K. R. Muske, and J. A. Rossiter. Conditions for which linear MPC converges to the correct target. *Journal of Process Control*, 20(10):1243 – 1251, 2010. ISSN 0959-1524. doi: http://dx.doi.org/10.1016/j.jprocont.2010.09.001. URL http://www.sciencedirect.com/science/article/pii/S0959152410001812.

S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis And Design*. John Wiley, 2005. ISBN 9780470011683. URL http://books.google.dk/books?id=I6lsQgAACAAJ.

P. Tøndel, T. A. Johansen, and A. Bemporad. Evaluation of Piecewise Affine Control via Binary Search Tree. *Automatica*, 39(5):945–950, May 2003.

A. van Schijndel. Integrated heat, air and moisture modeling and simulation in hamlab. In *IEA Annex 41 working meeting, Montreal, May*, 2005.

J. Široký, F. Oldewurtel, J. Cigler, and S. Prívara. Experimental analysis of model predictive control for an energy efficient building heating system. *Applied*

Energy, 88(9):3079 – 3087, 2011. ISSN 0306-2619. doi: 10.1016/j.apenergy.2011.03.009.

H.P. Williams. *Model Building in Mathematical Programming*. John Wiley & Sons, Third Edition, 1993.

X. Wu, J. Shen, Y. Li, and Lee. K. Data-driven modeling and predictive control for boiler–turbine unit using fuzzy clustering and subspace methods. *{ISA} Transactions*, 53(3):699 – 708, 2014. ISSN 0019-0578. doi: http://dx.doi.org/10.1016/j.isatra.2013.12.033. URL http://www.sciencedirect.com/science/article/pii/S0019057813002437.

Y. Zhang, B. Xian, and S. Ma. Continuous robust tracking control for magnetic levitation system with unidirectional input constraint. *IEEE Transactions on Industrial Electronics*, 62(9):5971–5980, Sept 2015. ISSN 0278-0046. doi: 10.1109/TIE.2015.2434791.

APPENDIX A

Author's Publications

This is a list[1] of scientific publications on which I have participated during my PhD study. The list is structured based on the categorization of Slovak Accreditation Committee[2]. Total no. of citations: 10.

Accreditation Category A; ADC - Articles in journals:

1. Klaučo, M., Kvasnica, M.: Control of a boiler-turbine unit using MPC-based reference governors. *Applied Thermal Engineering*, vol. 110, pg. 1437–1447, 2017 (IF: 3.043), citations: 1.

2. Drgoňa, J., Klaučo, M., Janeček, F., Kvasnica, M.: Optimal control of a laboratory binary distillation column via regionless explicit MPC. *Computers & Chemical Engineering*, vol. 96, pg. 139–148, 2017. (IF: 2.581)

3. Klaučo, M., Kalúz, M., Kvasnica, M.: Real-time implementation of an explicit MPC-based reference governor for control of a magnetic levitation system. *Control Engineering Practice*, vol. 60, pg. 99–105, 2017. (IF: 1.830)

4. Oravec J., Klaučo, M., Kvasnica, M., Lofberg, J., Computationally Tractable Formulations for optimal Path Planning with Interception of Targets' Neighborhoods, Journal of Guidance, Control, and Dynamics, pp. 1221–1230, 2017 (IF: 1.291)

5. *Holaza, J., Klaučo, M., Oravec, J., Drgoňa J., Kvasnica, M. and Fikar M.: MPC-Based Reference Governor Control of a Continuous Stirred-Tank Reactor. Computers & Chemical Engineering, vol. 96, pg. 139–148, 2017. (after 1st review, IF: 2.581)*

[1] Items in italics are not yet published, they are either accepted or submitted

[2] A minimum requirement for obtaining the PhD degree is to co-author 1 publication of category A, and 2 publications of category B

Accreditation Category A; ADM - Articles in journals:

6. Klaučo, M., Blažek, S., Kvasnica, M.: An Optimal Path Planning Problem for Heterogeneous Multi-Vehicle Systems. *International Journal of Applied Mathematics and Computer Science*, č. 2, vol. 26, pg. 297–308, 2016. (IF: 1.037), citations: 1.

Accreditation Category A; AFC - IFAC Proceedings:

7. Klaučo, M., Drgoňa, J., Kvasnica, M., Di Cairano, S.: Building Temperature Control by Simple MPC-like Feedback Laws Learned from Closed-Loop Data. In Preprints of the 19th IFAC World Congress Cape Town (South Africa) August 24 - August 29, 2014, pg. 581–586, 2014. *(personally delivered talk)*

8. *Holaza, J., Klaučo, M., and Kvasnica, M.: Solution techniques for multi-layer MPC-based control strategies (accepted). In Preprints of the 20th IFAC World Congress, France, pages –, 2017.*

9. *F. Janeček, M. Klaučo, M. Kalúz and M. Kvasnica.: OPTIPLAN: A Matlab Toolbox for Model Predictive Control with Obstacle Avoidance. (accepted). In Preprints of the 20th IFAC World Congress, France, pages –, 2017.*

Accreditation Category B; AFC - Conference Proceedings:

10. Jelemenský, M., Klaučo, M., Paulen, R., Lauwers, J., Logist, F., Van Impe, J., Fikar, M.: Time-Optimal Control and Parameter Estimation of Diafiltration Processes in the Presence of Membrane Fouling. In 11th IFAC Symposium on Dynamics and Control of Process Systems, including Biosystems, vol. 11, pg. 242–247, 2016.

11. Drgoňa, J., Janeček, F., Klaučo, M., Kvasnica, M.: Regionless Explicit MPC of a Distillation Column. In European Control Conference 2016, Aalborg, Denmark, pg. 1568–1573, 2016.

12. Oravec, J., Klaučo, M., Kvasnica, M., Löfberg, J.: Optimal Vehicle Routing with Interception of Targets' Neighbourhoods. In European Control Conference 2015, Linz, Austria, pg. 2538–2543, 2015.

13. Drgoňa, J., Klaučo, M., Kvasnica, M.: MPC-Based Reference Governors for Thermostatically Controlled Residential Buildings. In 54rd IEEE Conference on Decision and Control, Osaka, Japan, vol. 54, 2015.

14. Klaučo, M., Blažek, S., Kvasnica, M., Fikar, M.: Mixed-Integer SOCP Formulation of the Path Planning Problem for Heterogeneous Multi-Vehicle Systems. In European Control Conference 2014, Strasbourg, France, pg. 1474–1479, 2014. *(personally delivered talk)*, citations: 1.

15. Klaučo, M., Kvasnica, M.: Explicit MPC Approach to PMV-Based Thermal Comfort Control. In 53rd IEEE Conference on Decision and Control, Los Angeles, California, USA, vol. 53, pg. 4856–4861, 2014. *(personally delivered talk)*, citations: 1.

16. Drgoňa, J., Kvasnica, M., Klaučo, M., Fikar, M.: Explicit Stochastic MPC Approach to Building Temperature Control. In IEEE Conference on Decision and Control, Florence, Italy, pg. 6440–6445, 2013. *(personally delivered talk)*, citations: 4.

17. Klaučo, M., Kvasnica, M.: Modeling of Networked Systems in Simulink. Editors: Ivan Taufer, Daniel Honc, Milan Javurek, In Proceedings of the 10th International Scientific - Technical Conference Process Control 2012, University of Pardubice, Kouty nad Desnou, Czech Republic, 2012.

Accreditation Category B; AEC - Conference Proceedings:

18. Nehéz, M., Bernát, D., Klaučo, M.: Comparison of Algorithms for Near-Optimal Dominating Sets Computation in Real-World Networks. Editors: B. Rachev, A. Smrikarov, In Proceedings of the 16th International Conference on Computer Systems and Technologies, Association for Computing Machinery (ACM), Dublin, Ireland, pg. 199–206, 2015.

Accreditation Category B; AEC - Conference Proceedings:

19. *Holaza, J.; Valo, R.; Klaučo, M.: A Novel Approach of Control Design of the pH in the Neutralization Reactor. Editors: M. Fikar and M. Kvasnica, In Proceedings of the 20th International Conference on Process Control, Slovak Chemical Library, Štrbské Pleso, Slovakia, 2017.*

20. *Janeček, F.; Klaučo, M.; Kvasnica, M.: Trajectory planning and following for UAVs with nonlinear dynamics. Editors: M. Fikar and M. Kvasnica, In Proceedings of the 20th International Conference on Process Control, Slovak Chemical Library, Štrbské Pleso, Slovakia, 2017.*

21. *Ingole, D.; Drgoňa, J.; Kalúz, M.; Klaučo, M.; Bakošová, M.; Kvasnica, M.: Model Predictive Control of a Combined Electrolyzer-Fuel Cell Educational Pilot Plant. Editors: M. Fikar and M. Kvasnica, In Proceedings of the 20th*

International Conference on Process Control, Slovak Chemical Library, Štrbské Pleso, Slovakia, 2017.

22. Kalúz, M., Klaučo, M., Kvasnica, M.: Real-Time Implementation of a Reference Governor on the Arduino Microcontroller. Editors: M. Fikar and M. Kvasnica, In Proceedings of the 20th International Conference on Process Control, Slovak Chemical Library, Štrbské Pleso, Slovakia, pg. 350–356, 2015. *(personally delivered talk)*, citations: 2.

23. Drgoňa, J., Klaučo, M., Valo, R., Bendžala, J., Fikar, M.: Model Identification and Predictive Control of a Laboratory Binary Distillation Column. Editors: M. Fikar and M. Kvasnica, In Proceedings of the 20th International Conference on Process Control, Slovak Chemical Library, Štrbské Pleso, Slovakia, 2015.

24. Klaučo, M., Jelemenský, M., Valo, R., Fikar, M.: Identification and Control of a Laboratory Distillation Column. Editors: J. Cigánek, Š. Kozák, A. Kozáková, D. Rosinová, In Medzinárodná konferencia Kybernetika a Informatika '14, Nakladateľstvo STU v Bratislave, Oščadnica, vol. 27, 2014. *(personally delivered talk)*

25. Klaučo, M., Poulsen, N. K., Mirzaei, M., Niemann, H. H.: Frequency Weighted Model Predictive Control of Wind Turbine. Editors: Fikar, M., Kvasnica, M., In Proceedings of the 19th International Conference on Process Control, Slovak University of Technology in Bratislava, Štrbské Pleso, Slovakia, pg. 347–352, 2013. *(personally delivered talk)*

Accreditation Category B; AFG - Abstracts on Conference Proceedings:

26. Ingole, D., Drgoňa, J., Kalúz, M., Klaučo, M., Bakošová, M., Kvasnica, M.: Explicit Model Predictive Control of a Fuel Cell. In The European Conference on Computational Optimization, Leuven, Belgium, vol. 4, 2016.

GAI - Technical Reports:

27. Klaučo, M.: Modeling of the Closed-loop System with a Set of PID controllers. Radlinského 9, 812 37 Bratislava, 2016.

APPENDIX B

Curriculum Vitae

Professional Experience:

- October 2015 – March 2016
  Position: *Visiting Scholar*
  Institution: *University of California, Berkeley, California, USA*
  Research: *Application of Optimization in Obstacle Avoidance Problems*
  Supervisor: *prof. Franceso Borrelli, PhD*

- September 2013 – present
  Position: *PhD Candidate*
  Institution: *Slovak University of Technology in Bratislava, FCFT, IAM*
  Research: *MPC-Based Reference Governors: Theory and Application*
  Supervisor: *Assoc. Prof. Michal Kvasnica, PhD*

- September 2013 – present
  Position: *Assitant Teacher*
  Institution: *Slovak University of Technology in Bratislava, FCFT, IAM*
  Courses: *Model Predictive Control, Theory of Automation Control, Process Control, Optimization, Process Optimization*

- September 2012 – August 2013
  Position: *Project Application Engineer*
  Company: *Invensys Systems (Slovakia), Bratislava*
  Project: *China Nuclear Project (Implementation of Core Control System for 440MW Nuclear Reactor in Tricon Software)*

Education:

- June 2013
  Degree: *MSc. in Automation and Informatization in Chemistry and Food Industry*

Institution: *Slovak University of Technology in Bratislava, FCFT, IAM*

- August 2012
  Degree: *MSc in Automation and Control at DTU Elektro*
  Institution: *Denmark University of Technology*

- July 2010
  Degree: *BSc. in Automation and Informatization in Chemistry and Food Industry*
  Institution: *Slovak University of Technology in Bratislava, FCFT, IAM*

Scholarships, Honors & Awards:

- 2016 – Merit Scholarship at Slovak University of Technology
- 2015 – Traveling Grant from Nadácia Tatra Banky, Slovakia
- 2014 – Merit Scholarship at Slovak University of Technology
- 2012 – National Scholarship Program (SAIA, Slovakia)
- 2010 – Erasmus Scholarship for abroad study – Denmark
- 2010 – Dean's award for exceptional studies, Slovak University of Technology in Bratislava
- 2009 – 2$^{\text{nd}}$ place at Student Conference at FCFT, STU in Bratislava
- 2007 – Merit Scholarship at Slovak University of Technology

Publishing Records:

- 2017 – 4 journal papers, 4 conference papers
- 2016 – 1 journal paper, 2 conference papers, 1 talk delivered at ECC'16 Tutorial session
- 2015 – 4 conference papers, 1 talk delivered at PC'15
- 2014 – 3 conference papers, 3 talks delivered (ECC'14, IFAC'14, and CDC'14)
- 2013 – 2 conference papers, 2 talks delivered (PC'13, CDC'13)

- 2012 – 2 conference papers

Participation on Research Grants:

- Internal STU research funding - 2017: Advanced Optimal and Safety Oriented Control of Energy-Intensive Processes
- APVV-15-0007 - Optimal Control for Process Industries
- Internal STU research funding - 2016: Complex Predictive Control of Energy-Demanding Chemical Processes
- APVV SK-CN-2015-0016 – CN-SK cooperation: Robust Model Predictive Control Meets Robotics
- VEGA 1/0403/15 - Verifiably Safe Optimal Control
- VEGA 1/0053/13 - Optimal Process Control
- APVV 0551-11 - Advanced and effective methods of optimal process control
- VEGA 1/0973/12 - Control of Processes with Uncertainties in Chemical Technology and Biotechnology
- VEGA 1/0095/11 - Model Predictive Control on Platforms with Limited Computational Resources

APPENDIX C

Resumé

Predkladaná dizertačná práca sa venuje rozšírenej aplikácií prediktívneho riadenia, ktorou je použitie tohto optimálneho riadenia na riadenie vnútornej slučky, ktorá už obsahuje stabilizujúci regulátor. Tento stabilizujúci regulátor je člen riadiacej slučky, ktorý je zodpovedný za poskytovanie akčných zásahov do akčného člena. Tento akčný zásah je určený na základe referencie, ktorá však je poskytnutá optimalizáciou. Zo zahraničnej literatúry je tento typ riadiacej stratégie známy pod pojmom *optimization-based reference governor*. Konkrétnejšie sa však v tejto práci venujeme prípadu, kedy tento nadradený riadiaci člen je vo forme prediktívneho regulátora (z angličtiny – model predictive control (MPC)). Vtedy budeme hovoriť o *MPC-based reference governors* (MPC-RG), resp. *Supervízory na báze MPC*. Pre bližšiu ilustráciu, si pozrime obr. 1.1, na ktorom je znázornená schéma takéhoto pokročilého riadenia. Medzi hlavné výhody takejto stratégie riadenia patria

- zvýšenie bezpečnosti prevádzky,
- zabezpečenie dodržania technologických ohraničení optimálnym spôsobom,
- systematické znižovanie energetických nárokov a zvyšovanie produkcie.

Okrem vyššie spomenutých výhod, má použitie MPC-RG ešte jednu dôležitú výhodu. Tou je fakt, že v prípade nasadzovania tejto stratégie, nie je potrebné meniť súčasnú riadiacu infraštruktúru a teda je možné ponechať už existujúce regulačné obvody.

Vzhľadom na fakt, že MPC supervízory sú optimálne riadiace stratégie, tak v tejto dizertačnej práci ako prvé uvedieme základné typy optimalizačných problémov (kapitola 2), ktoré sa neskôr využijú pri formulovaní MPC riadiacich schém. Bližšie si predstavíme problémy lineárneho a kvadratického pro-

gramovania. Tieto problémy je potom možné efektívne riešiť pomocou rôznych profesionálnych nástrojov ako sú softwarové balíky GUROBI[1], CPLEX[2], alebo MOSEK[3]. Ďalej predstavujeme optimalizačné problémy zmiešaného celočíselného programovania, ktoré sa taktiež využívajú pri formulovaní MPC supervízorov pre špecifické kategórie uzavretých riadiacich obvodov. Posledná časť kapitoly o optimalizácii je venovaná parametrickému riešeniu kvadratických optimalizačných problémov. Toto parametrické programovanie sa používa v prípade, že MPC supervízor je potrebné implementovať na zariadenia s obmedzenou výpočtovou kapacitou, čo znemožňuje použitie balíkov ako je GUROBI a.i.

V rámci prehľadu súčasnej literatúry uvádzame aj teoretické princípy fungovania MPC regulátorov, ktorým je venovaná kapitola 3. V tejto kapitole rozoberáme základné formulácie MPC riadenia, ich štruktúru a možnosti rozšírenia o modelovanie porúch a Kalmanov filter za účelom poskytnutia riadenia na žiadanú hodnotu.

Teoretické prínosy tejto dizertačnej práce sú opísané v kapitole 4. V začiatku kapitoly je načrtnutý všeobecný problém riadenia pomocou MPC supervízora v rovnici (4.1). Jedna časť teoretických výsledkov sa zameriava práve na reformuláciu tohto všeobecného optimalizačného problému, na takú formáciu, ktorú vieme efektívne riešiť pomocou dostupných softwarových balíkov. Konkrétne ide o matematické modelovanie uzavretej slučky, ktorá je vyjadrená pomocou rovníc (4.1b), (4.1c) a (4.1d). V sekciách kapitoly 4, rozoberieme nasledovné prípady uzavretých regulačných obvodov pozostávajúcich z:

1. **PID regulátorov**, predstavené v podkapitole 4.1, ktorej výsledky a aplikácie su publikované v [Klaučo et al., 2017, Klaučo and Kvasnica, 2017],
2. **on/off regulátorov**, predstavené v podkapitole 4.2, odpublikované v [Drgoňa et al., 2015],
3. **lokályuch MPC regulátorov**, bližšie rozvedené v podkapitole 4.3, a publikované v Holaza et al. [2017].

V každom z týchto troch prípadov, najskôr odvodíme model uzavretej regulačnej slučky, čiže určíme rovnice (4.1b), (4.1c) a (4.1d). Na základe týchto modelov potom zostavíme jednotlivé MPC supervízory. V podkapitole 4.1, ukazujeme ako pretransformujeme zákon riadenia daný PID regulátorom v tvare

[1] www.gurobi.com
[2] www-01.ibm.com/software/commerce/optimization/cplex-optimizer/
[3] www.mosek.com

prenosovej funkcie do stavového opisu, čo nám následne umožní naformulovať MPC-RG problém ako rozšírené MPC riadenie, podobne ako to bolo uvedené v kapitole 3. Výsledný MPC-RG optimalizačný problém je potom v tvare kvadratického optimalizačného problému (z angličtiny - quadratic programming (QP)) alebo vo forme lineárneho programovania (z angličtiny - linear programming (LP)), podľa toho aký typ kvalitatívneho kritéria si zvolíme. Sekcia 4.2 sa venuje modelovaniu uzavretého riadiaceho obvodu, ktorý pozostáva z riadeného procesu a z on/off regulátora. V prípade on/off regulátora je model uzavretej slučky vyjadrený pomocou binárnych premenných, ktoré výrazne ovplyvňujú celú štruktúru MPC supervízora. Výsledný optimalizačný problém, ktorý reprezentuje MPC-RG stratégiu je navrhnutý ako zmiešaný celočíselný optimalizačný problém (z angličtiny - mixed-integer (MI) problem). V závislosti od voľby kvalitatívneho kritéria riadenia, t.j. účelovej funkcie, je tento problém buď klasifikovaný ako MILP, v prípade lineárnej účelovej funkcie alebo MIQP, v prípade kvadratickej účelovej funkcie. Oba typy prípadov zmiešaných celočíselných optimalizačných problémov sa dajú efektívne riešiť pomocou dostupných profesionálnych softwarových balíkov ako je GUROBI, CPLEX alebo MOSEK. Limitujúcim faktorom na rýchle výpočty, na rozdiel od QP/LP problémov, je v prípade MI problémov počet binárnych premenných. Zložitosť riešenia MI problémov v princípe rastie exponenciálne s počtom binárnych premenných. Podkapitola 4.3 rozoberá návrh MPC supervízora pre uzavreté regulačné obvody vprípade, že primárny regulátor je jednoduchý MPC regulátor. V tomto prípade uvažujeme scenár, že máme niekoľko uzavretých regulačných obvodov, ktoré navzájom nie sú prepojené, avšak zdieľajú rovnaké zdroje, ktorý reprezentuje vstupný signál do týchto jednotlivých procesov. Tento prípad je štandardným scenárom v priemyselných aplikáciách. V tomto prípade modelovanie uzavretej slučky je samotným optimalizačným problémom, ktorý keď skombinujeme s MPC-RG prístupom, dostaneme takzvanú viacúrovňovú optimalizáciu (z angličtiny – bilevel optimization). Tieto typy optimalizačných úloh sa nedajú triviálne riešiť. V tejto práci ukážeme dva prístupy, ako sa vysporiadať s týmto typom úloh, aby sme dostali relatívne ľahko riešiteľný problém. Prvý prístup predpokladá rozpísanie vnútorného optimalizačné=ho problému pomocou Karush-Kuhn-Tuckerovych (KKT) podmienok optimality. Ak sa aplikuje tento spôsob, vo výsledku dostaneme jeden optimalizačný problém, ktorý pozostáva z nelineárnych hraničení reprezentujúcich KKT systém. Tieto KKT podmienky sa dajú následne pretransformovať pomocou binárnych premenných do lineárnych ohraničení. Vo výsledku teda dostaneme optimalizačný problém v tvare zmiešaného-celočíselného programovania, ktorý už vieme riešiť efektívnejšie. Druhý prístup predpokladá, že vnútorný MPC problém vyriešime najprv explicitne, t.j. dostaneme analytické riešenie problému v tvare PWA funkcie, ktorú následne opäť pomocou binárnych premenných zapíšeme ako model do MPC-RG problému. Opäť dostávame MIQP optimalizačný problém.

V druhej časti tejto dizertačnej práce sa bližšie pozrieme na aplikáciu MPC-RG stratégií. V kapitole 5 ukážeme návrh MPC-RG stratégie na riadenie levitujúcej guličky v magnetickom poli. V tomto prípade používame aj parametrické riešenie tohto MPC-RG problému aby sme ho boli schopní implementovať na mikročipe. Experimentálne výsledky ukazujú, že zavedením MPC-RG stratégie sa výrazne zlepší regulačný pochod, a zároveň ukazujeme akým optimálne dodržanie fyzikálnych ohraničení, ako sú limity na akčný zásah, alebo limit na polohu levitujúcej guličky. Ďalšou aplikáciou MPC-RG prístupu (kapitola 6), je zlepšenie regulačných pochodov energeticky náročného technologického zariadenia, ktorým je parná turbína. Primárne je táto turbína riadená troma PID regulátormi. Ak však navrhneme MPC-RG stratégiu, tak sme schopní zlepšiť výkonnosť až o 30%, pri rovnakých energetických nárokoch. V poslednej kapitole 7 sa venujeme zlepšovanie tepelného komfortu pomocou MPC supervízora, ktorý riadi termostat. Termostat je v princípe on/off regulátor. Taktiež zavedením MPC supervízora ukazujeme, že energetická úspora pomocou tejto stratégie dosahuje až 17%.

V závere práce sú zhodnotené teoretické prínosy ako aj aplikačné výsledky. Taktiež sú v závere dizertačnej práce načrtnuté aj budúce témy na ďalší výskum. Medzi tie hlavné body patrí modelovanie iných typov uzavretých slučiek a prepojenie MPC-RG stratégií s prístupmi strojového učenia.